STING OPERATIONS ON CORRUPTION NETWORKS

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Abstract. Networks have become quite commonplace as tools for modeling crime, especially in the past two decades. However, they are still relatively new tools for analyzing bribery and corruption. In this thesis, I attempt to analyze the effectiveness of 'sting operations' by external law enforcement agencies on corrupt hierarchies. We find that the best strategy to achieve complete deterrence in a hierarchy depends on the comparative costs of placing sting agents in different levels, as well as the comparative severity of punishment for bribe-givers and takers. In the most realistic scenario, however, we find that the most efficient strategy is to target alternate layers, at least in hierarchies with an odd-number of levels.

Keywords— Hierarchy, Sting Operations, Bribery, Deterrence

Contents

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1. Introduction

Corruption is one of the biggest challenges facing developing countries and has been a topic of much research for the past 30 years. In particular, India, where I am from, has consistently ranked in the bottom half of the World Corruption Index Rankings since their inception, something that is also apparent from Figure 1. More generally, research by Spyromitros & Panagiotidis (2022) has found that for a 1% increase in corruption, as measured by various indices, there is a 0.2% decrease in economic growth rate. The effects on income inequality are similar but more complex; higher wage differentials between private and public sectors fuel corruption [], forming a positive feedback loop.

Intuitively, the higher prevalence of corruption in poorer countries can be explained by the lower opportunity cost of giving up legal income, especially for government officials. Research by Van Rijckeghem & Weder (2001) suggests that well-paid government officials are more wary of accepting bribes. On the other hand, Abbink (2000) suggests that the opportunity cost is moral, and government officials might find it less unacceptable to accept bribes if they are poorly paid. Figure 2 depicts the severity of corruption around the world.

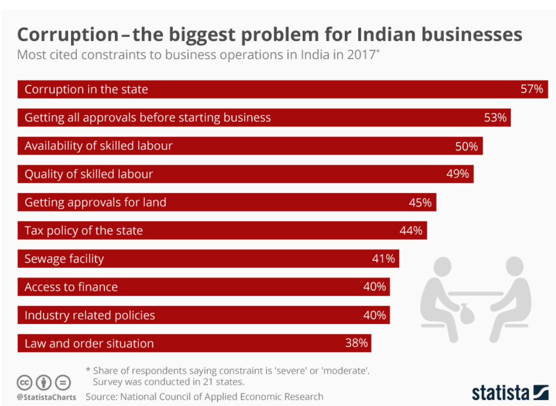

Figure 1. Challenges Facing Indian Businesses

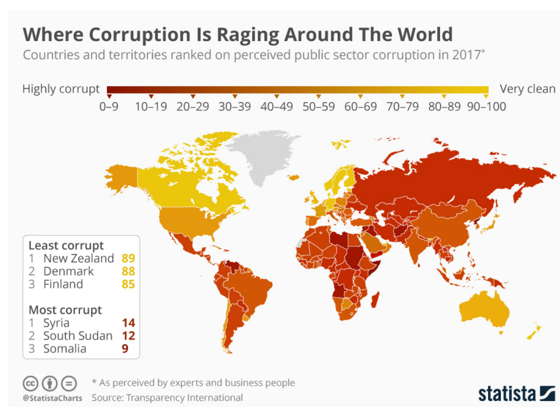

Figure 2. Corruption Around the World

Most research on corruption, however, has been empirical; examining its effect on economic growth and development in general and proposing policy that may help prevent it. In my project, I wish to study corruption, or rather its prevention, theoretically; investigating organizational structures and the effects of *sting operations* on them. The theoretical approach is partly motivated by the lack of public knowledge about the details of sting operations, but more so by the fact that optimization is always easier to study in a model than real-life examples. This is even more important because our goal is to study deterrence rather than conviction, which is impossible to even measure in the real world, but has been reasoned to be cheaper than punishment, mainly because we save the cost of punishing, such as building prisons (Chalfin & McCrary, 2017). We will be analyzing under what conditions sting operations most effectively deter corruption, and how to conduct them most efficiently.

What then is a sting operation? Let us say that an external law enforcement agency suspects that there is criminal activity taking place within an organization; however, it is hard to confirm this suspicion through external investigation. To investigate their suspicion, the law agency sends an operator undercover into this organization, who tries to catch its members by deceiving them into committing a crime. The Narada Operation was an important sting operation in West Bengal, India that dealt primarily with bribery and exposed large parts of the top brass of the current ruling party, and is one of the primary inspirations behind this study. Logically, sting operations deter in two ways, through the fear that a sting agent will record a bribe, and the punishment that will occur when they do. In this project, we will primarily be focusing on optimizing the probability of recording bribes as deterrence rather than the severity of punishment.

Of course, this analysis can be done for any organizational structure, but we are going to be analyzing network structures, specifically hierarchies. A hierarchical structure is the chain of command within an organization that begins with the top executives and then extends downwards. One can represent a hierarchy as a directed tree, which is a graph where any two vertices are connected by exactly one path. The top levels hold the most authority but tend to have fewer members. The employees near the bottom levels have less authority but tend to have greater numbers. Hierarchies are the most common organizational structure, and therefore it is logical to confine our analysis to them for now. Moreover, in the past, multiple sting operations have been successfully carried out on hierarchies, most notably by Frank Serpico on the NYPD in the 1970s, revealing that policemen from all layers received favors from criminals and lower-ranked officers, while also covering up crimes and each other's bribe-taking. This story further implies that corruption is often systematically organized into hierarchies, making them a good choice for our analysis. Here I give an example of two hierarchies, where the arrows show which supervisor has power over which subordinates.

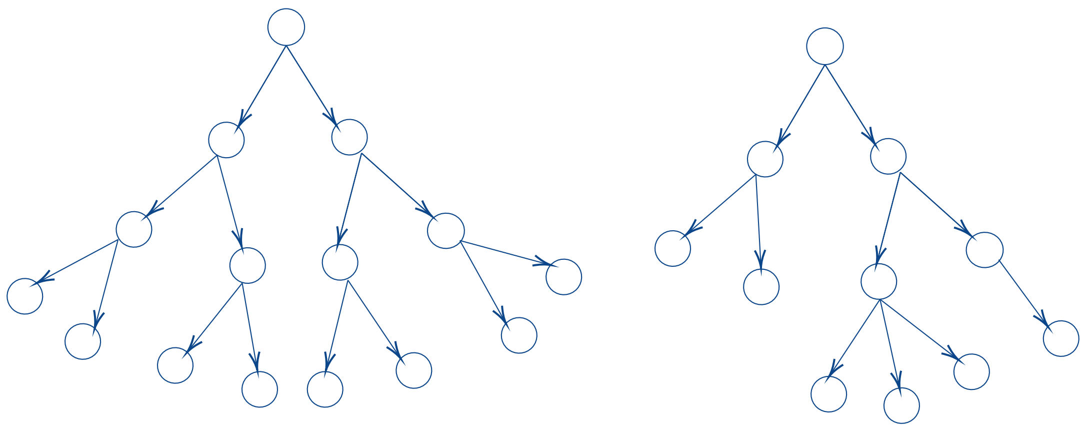

Figure 3. Examples of Hierarchies

Following this, the thesis is organized as follows: In Section 2, I will be discussing in-depth the prior literature and how the methods used there are relevant to my study. I will also be presenting results from various papers on the Economics of Crime to justify some of the assumptions made to simplify our model. In Section 3, I will be detailing the model, introducing the necessary notation, and including the cost structures and the underlying games.

Moving on, in Section 4, I will be discussing the central question we will be answering in this project, the complete deterrence of bribery in a hierarchy for the lowest possible cost. Section 5, will dive into an interesting extension of that question, even though we will not have time to fully answer that. Finally, we will be discussing the conclusions for this research project, as well as some interesting next steps.

I would like to note here that throughout this thesis I am going to be using the terms **severity of punishment** and **amount of penalty** interchangeably. Similarly, I will also be using the terms **level** and **layer**, in the context of a hierarchy, equivalently.

2. Background Literature

This particular area of research is quite new; only Fan (2022) directly models bribery as a network. However, the objective of that paper is to find the optimal strategy for giving and taking bribes in the presence of monitoring, rather than to find optimal solutions. In her paper, Fan analyzes two different network structures, hierarchies, and two-layered networks (mainly motivated by the possibility of corrupt law enforcement agents, a feature missing from my model), comparing the resulting optimal strategies.

As this is where I drew inspiration for my model, it is valuable to discuss the details of this model. Subordinates offer bribes to their supervisors to gain favor and advantages. A player is caught by an external law enforcement agency if they are reported by a supervisor or discovered by the law agency through accepting a subordinate's bribe with independent probability $s \in (0, 1)$. It is important to note here that the supervisor is not a sting agent in this model, each supervisor, depending on the amount of the bribe and other factors, may choose to report a subordinate, again this is not a feature in our model. If detected, the player has to surrender all bribes and pay a large cost as a fine. There are a few interesting results here that are partially relevant to our model and the next steps for the research project (not in this honors project). First, the risk of being caught while accepting a subordinate's bribe increases with every bribe for hierarchies, but falls with every bribe in two-layer networks. Second, if the network becomes denser, that is more players are introduced at every level of the network, then the optimal number of bribes falls for all players. Unfortunately, we do not have the time to talk about network structures yet, but I will be briefly discussing the interest that we might have in degree if we relax certain assumptions.

More generally, however, there exists significant research on optimal network formation for criminal organizations, given their twin goals of efficient communication and detection avoidance. Baccara & Bar-Issac (2008) particularly study optimal network formation in terrorist organizations where each player chooses which of his fellow terrorists to reveal his identity to best achieve those goals. While disclosing personal information improves group efficiency, it makes an agent more vulnerable to external threats. The external authority here has a fixed budget that they can devote to detecting the N players in the criminal organization. While this paper has a similar focus to that of our project, it differs in structure: First, while they consider completely free network formation, we fix the network and examine the effects of placing operators in the network. Second, again, it tries to optimize the structure of a criminal organization, whereas we are attempting to reduce crime given a structure. As for results, this paper finds that the most efficient structure, for information sharing, it finds, are either binary cells or a hierarchy with the information hub being a singleton or a binary cell, and for N-linked players is a hierarchy with the player with the lowest probability of detection at the top of the hierarchy. In our project, we only consider hierarchies with singletons at the top, and by construction, the player at the top has the lowest chance of detection. Finally, Acemoglu, Malekian & Ozdaglar (2016) have a similar focus in their paper analyzing a situation where a network of agents is threatened by a cyberattack. Here, they examine an exogenous network structure, similar to what we are dealing with, but focus on how agents invest in self-immunity.

Our model is also going to be informed by the substantive work on the economics of crime. The primary paper in this area that we are going to be looking at is also one of the first ever written, "Crime and Punishment: An Economic Approach", by Becker (1968). There are certain differences between our model and Becker's approach to analyzing crime and punishment, mainly in the fact that he jointly determines both the probability of punishment and its severity, while we are only focusing on the likelihood of conviction as a deterrent. This approach certainly has its benefits: Saha & Poole (2000) discover that given the objective of minimizing the probability of transgression and monitoring costs, optimal punishments are 60% lower if determined endogenously rather than exogenously (i.e. making the bribe amount into part of the strategies of both parties to the bribe transaction such that the monitoring probability and the penalty level are jointly determined).

They also demonstrate that, if the choice of penalty were to be determined by the external agency, the chosen penalty level would be lower than that socially desired.

However, we do not follow this method for two main reasons. Firstly, endogenizing costs makes the model far more complicated to solve and secondly, realistically, the size and probability of punishments are rarely decided together as they are under the purview of different government departments. Despite this, there are many relevant takeaways from this paper. One of Becker's most important findings was that increasing the probability of conviction decreases the probability of crime by a greater amount than increasing the severity of crime. Therefore, given that we are working primarily on deterrence, it makes sense that between the two we are focusing on optimizing the probability of conviction. Even more importantly, Becker finds that the optimality condition for the punishment is that it be proportional to the "marginal harm caused" by an individual, which is an important assumption in the context of our model and will be elaborated upon later.

The other area of economics of crime research we are interested in, mainly because it helps us justify one of our findings, is the relationship between the severity of punishment and crime rates, where there has been both theoretical and empirical research. There is abundant empirical evidence, such as from a survey conducted by Levitt & Miles (2007), that increases in punishment severity deter crime. At the same time, Smith (1999) through cross-country analyses of crime and punishment has shown that the relationship is more complex and it is possible to have lower crime without heftily increasing the threat of punishment. In particular, the United States has a much higher incarceration rate compared to other developed countries, but that has not translated into a correspondingly lower crime rate, even after controlling for other factors that affect crime. Friehe & Miceli (2017) explain this paradox by suggesting that harder criminal sanctions will increase the efforts made by offenders to avoid justice, thereby leading to an ambiguous relationship between punishment severity and the crime rate. Switching gears, On a related note, Kleiman & Kilmer (2009) find, through simulation, that there exist two different equilibria, a high crime and a low crime one, and there exists a "tipping" point, which makes it possible to move from one equilibrium to the other. In that case, temporary increases in punishment capacity can lead to lasting changes in violation rates. These findings help us explain the spectacular results of some criminal-justice interventions using focused deterrence as Kessler & Levitt (1998) corroborate through their finding that the law requiring longer sentences has been effective in lowering crime. In the United States within three years, violent crimes covered by the law fell an estimated 8 percent. Seven years after the law changed, these crimes were reduced by a staggering 20 percent. These papers primarily instruct us in better understanding how to practically analyze and interpret the punishment outcomes we might get.

3. Model

3.1. Defining Costs and Network Structures.

At this stage, our structure is a regular hierarchy that we are fixing (it may be assumed that the number of players under a given supervisor is determined by nature, i.e. we do not really care for this case), so as to not allow the players in the game to respond by changing the hierarchy structure in order to evade at least some of the sting operators. Each player has at most one supervisor, so there is no co-supervision. There is a "power structure" assumption in place here; each player's supervisors can do everything that they can, but they can also do more. Therefore, to gain favor from them, lower-down players may bribe their supervisors. However, it is only possible to bribe your immediate supervisor. Bribe prices are naturally higher for higher-layer players, and if a player tries to bypass a layer the cost of a bribe is prohibitive.

Into this organization, the external law sends in sting operators. "Send" is a loose term. It could refer to both actually placing a covert external agent and faking his identity, or just turning someone who is already an employee. In many circumstances, the latter is more feasible or cheaper. In both cases, having an operator on a higher level of the hierarchy is more expensive. This makes intuitive sense, as in the first case it is harder to fake the credentials of a higher-up, as they are more under scrutiny, and in the second case because they are higher paid and they sacrifice more in bribes by "going honest". This model does not allow the possibility of turning people into agents by catching them giving or receiving bribes. We assume that the cost of placing an agent at each layer is independent of placing an agent in a different layer.

We model this situation as a sequential game. First, the law agency announces a p for each level of the hierarchy, which is the probability that any of the players in that level is a sting operator, so the supervisors assume that the operators are placed in a uniform random manner. This makes sense as our focus is deterrence. Furthermore, the law agency must uphold this commitment, as otherwise there are always incentives for the agency to deviate to 0. This is possible because after deterring bribery for one period, for the next period, the law enforcement agency has no incentive to keep the sting agents in place, which means that they will remove them, resulting in the bribery returning in the next period. Therefore to prevent this oscillating behavior we have the agency publicly announce the p so that they cannot change it. In the next period, each player may offer a bribe to their supervisor. Each player has an independent probability q of being a “bribe” type, which means that if not deterred they are going to bribe their supervisor, or in other words, they have bribery in their choice set. This enables us to have ‘honest’ players, who do not have bribery in their choice set, in an organization as well. We give an example below:

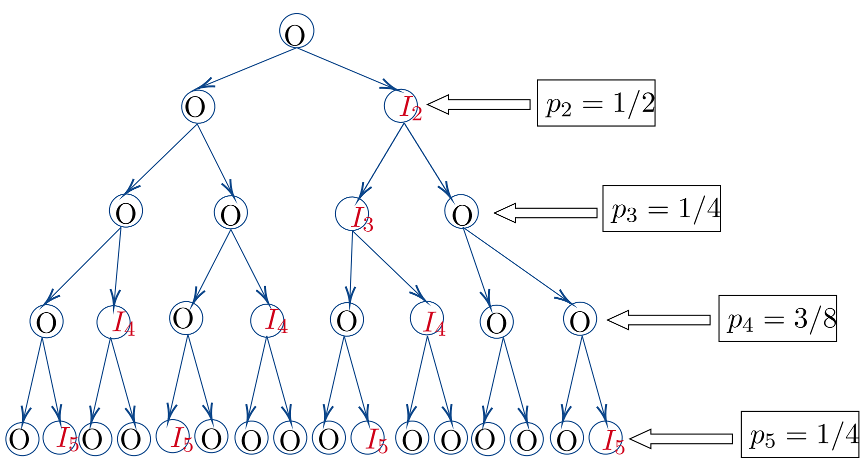

FIGURE 4. An Example of a Possible Model

Here, I stands for the sting operators and O stands for non-sting operator players (some of whom might be honest), and the arrows point the direction of influence, from each supervisor to their subordinates from whom they might receive bribes (it might have made more sense to make the arrows point in the direction the bribes are given, but I don't have any more time). Also note here that the higher the level of the hierarchy, the lower the number it is assigned, i.e. level i is higher than level $i+1$.

This also illustrates the cost to the law agency, which we are assuming to be linear for the sake of simplicity, is given by $I_2 + I_3 + 3 \times I_4 + 4 \times I_5$, where $I_2 > I_3 > I_4 > I_5$. More generally, for a given layer i, if I_i is the cost of placing a single agent in the layer, p_i is the announced $\mathbb{P}$ $(Sting)$ for the layer, and N_i is the number of players in the layer, then the total cost to the agency for the layer is $I_i \;\times\; p_i \;\times\; N_i$. So if there are n layers in our given hierarchy, then the total cost to the agency is $TC = \Sigma_{i=1}^{n}(I_i \;\times\; p_i \;\times\; N_i)$. We assume a geometric relationship in the costs of placing sting operators between adjacent layers. We can represent the geometric costs model as $I_i \cdot N_i = \alpha \cdot (I_{i+1} \cdot N_{i+1})$, for some $\alpha > 0$ and all i. This means we can distinguish between hierarchies with top-heavy costs, where it is more expensive to place agents at higher levels ($\alpha > 1$), and bottom-heavy costs where the opposite is true ($\alpha < 1$).

The other possible source of costs to the agency or the government could be the cost of punishing offenders. However, since in equilibrium, bribes are deterred, there should be no one who gets caught. On the other hand, if bribes are not deterred, then there is no reason even to send in sting operators, so again the cost of punishing is zero as no one will get caught.

The benefit to the law agency can be defined in a few ways, one of which is that for a given layer-pair i, j, if all bribery is prevented in the adjacent layer-pair (which must be the case since preventing giving from a layer also stops receiving from the layer above it), then the agency gets a fixed benefit $H_{i,j}$. The other way to determine benefits could be to estimate the number of bribes that would have taken place if there was no deterrence, which can be done with the help of β, and add the value of all of the prevented "bribes".

In this model exogenous variables are: 1) the size of the bribe at level i, E_i which at this point is the same for every player of a given level of the hierarchy, but varies between layers; 2) The penalty of being caught the size of the punishment at level i, C_i which again is the same for every player of a given layer of the hierarchy, but varies between layers; 3) Amount of advantage a successful bribe transaction creates overall between level i and $i+1$, V_i, which varies by level as well; 4) The probability that a player in the network is a type that gives bribes, q. Finally, we assume that the magnitude of the penalty is proportional to the size of the bribe that was being given or received, which in turn also implies that higher-up players get punished more if they are caught, as they require higher bribes, so for a n-levelled hierarchy, we have $E_1 \geq E_2 \geq \cdots \geq E_n$, $V_1 \geq V_2 \geq \cdots \geq V_n$, and $C_1 \geq C_2 \geq \cdots \geq C_n$.

Let us first consider a scenario, however, where there is no external law agency. Then, we can illustrate how exactly bribery will work in this model. p is going to be zero for every level, and C is effectively zero as well. Then during a single period, the "bribe types" will try giving their supervisors bribes, which they will accept and make a gain of E, and the bribe types get benefit $V - E$ with no downside, so they are best responding by offering bribes, due to the absence of a law enforcement agency. The question of the practicality of this scenario has been touched upon in the Introduction, especially in the case of Serpico and other NYPD-related cases. Next, we can begin to examine the effect of sting operations on the hierarchy.

3.2. **Defining the Games.**

We have two transaction "games" here, the bribe-giving game, and the bribe-receiving game. Before we go ahead and get into the games themselves in detail, let's clarify the abbreviations of the various variables (some of which might not appear right now, but will definitely be explained later) and move on to the diagram.

Players

- L - Law Enforcement
- O - Organization Official
- I - Sting Operator
- Empty Circle = Nature

Strategies

- S - Sting
- NS - No Sting
- B - Bribe
- NB - No Bribe
- A - Accept Bribe
- R - Reject Bribe

Types of Transaction

- G = Bribe Giving
- R = Bribe Receiving

Probabilities & Proportionality Constants

- $p_i = \mathbb{P}$ (Sting) (We might occasionally use the p_G or p_R notation to denote which one we are talking about, as we will in the explanation of the following game, but mostly it will be pretty evident)
- p_G^* = Cutoff p required to Stop Bribe Giving
- p_R^* = Cutoff p required to Stop Bribe Receiving
- $q = \mathbb{P}$ (Bribe | No Sting)
- $k_i = \mathbb{P}$ (Sting | Bribe)
- $\beta_G = 1$ if $(p_G < p_G^*)$, 0 otherwise
- $\beta_R = 1$ if $(p_R < p_R^*)$, 0 otherwise
- $\alpha > 1$ if the Hierarchy has Top-Heavy Costs, $\alpha < 1$ otherwise
- $0 < \pi < 1$ = The Bargaining Power of the Bribe Giver
- $0 < \rho_G < 1$ = The Proportionality Constant for which $C_i = \rho_G \cdot (V_i - E_i)$
- $0 < \rho_R < 1$ = The Proportionality Constant for which $C_i = \rho_R \cdot E_i$

Outcomes

- C_i = Magnitude of Penalty of Being Caught at Level i, We Assume $C_1 \geq C_2 \geq \cdots \geq C_n$. We have that the punishment for Giving Bribes is C^G, and the one for Receiving is C^R. However, these are going to be clear from context usually, and I will rarely use this notation.
- V_i = Amount of Value created by the Bribe Transaction between Level i and $i+1$, We Assume $V_1 \geq V_2 \geq \cdots \geq V_n$

- E_i = Amount of Profit from Receiving Bribe at Level i, We Assume $E_1 \geq E_2 \geq \cdots \geq E_n$
- $V_i - E_i$ = Amount of Advantage from Giving Bribe at Level i, this is always positive

Costs and Populations

- I_i = Cost of Placing one Sting Operator Level i
- N_i = Number of Total Players on Level i

Exogenous: E_i, C_i, V_i, I_i, N_i, q, ρ, α, π

Strategy: p_i

First, we take a closer look at the Bribe-Receiving game from the viewpoint of a single player for a single time period; as we are dealing with a single layer here, we will dispense with the i notation for all the variables:

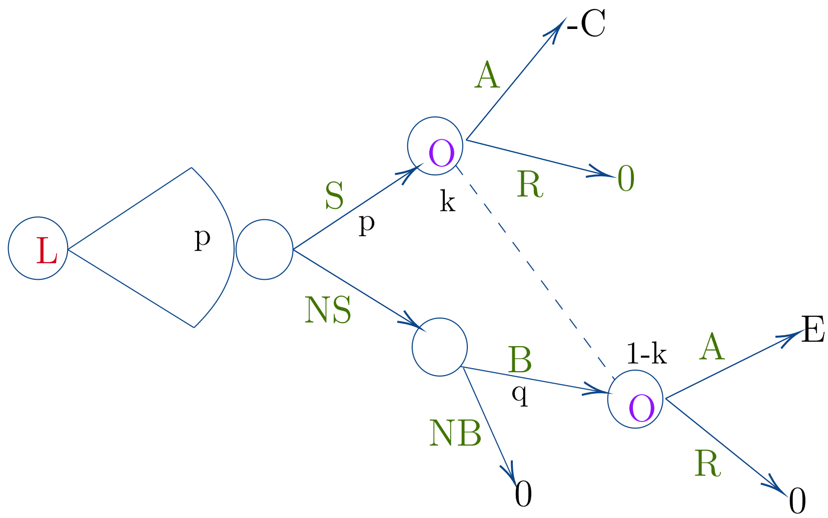

Figure 5. A Closer Look at the Receiving Game

The sequential nature of the game is clear from the diagram. As k is the same for every player in the layer, it can be thought of as a permeating "fear". A few more things need clarification. As we are looking at the game from the viewpoint of a single player from a given level, it is okay to fix the bribe amount E, as well as the penalty amount C. Also, notice the fact that the player does not know whether he is receiving bribes from a genuinely corrupt player or an operator. Now, our goal here is to prevent all bribe-receiving in a given layer. We know that the expected payoff for accepting a bribe for any player is:

$$\text{Payoff (Accepting Bribe)} = \text{E} \times (1- \mathbb{P}\ (\text{Sting} \mid \text{Bribe})\ ) - \text{C} \times \mathbb{P}\ (\text{Sting} \mid \text{Bribe}) = 0$$

$$\Rightarrow \text{E} \times (1- k) - \text{C} \times k = 0$$

So the Indifference Condition for Accepting bribes is: $k = \frac{\text{E}}{\text{E + C}}$

By Bayes' Theorem:

$$\begin{aligned}
\mathbb{P}\ (\text{Sting} \mid \text{Bribe}) &= \frac{\mathbb{P}\ (\text{Bribe} \mid \text{Sting}) \times \mathbb{P}\ (\text{Sting})}{\mathbb{P}\ (\text{Bribe})} \\
&= \frac{\mathbb{P}\ (\text{Sting})}{\mathbb{P}\ (\text{Bribe} \mid \text{Sting}) \times \mathbb{P}\ (\text{Sting}) + \mathbb{P}\ (\text{Bribe} \mid \text{No Sting}) \times \mathbb{P}\ (\text{No Sting})} \\
&= \frac{\mathbb{P}\ (\text{Sting})}{\mathbb{P}\ (\text{Sting}) + \mathbb{P}\ (\text{Bribe} \mid \text{No Sting}) \times [1-\mathbb{P}\ (\text{Sting})]} \\
k &= \frac{p_R}{p_R + q \times \beta_G \times [1-p_R]}
\end{aligned}$$

Then, the p_R that induces the indifference condition is $p_R^* = \frac{q\beta \times \text{E}}{q\beta_G \times \text{E} + C}$

As can be seen here, p_R^*, which is a direct indicator of the number of operators necessary to stop players from receiving bribes at a given level, is inversely proportional to C, the penalty for being caught. This is in some sense supported by the literature on crime, but it is also in some sense unrealistic, as here if C increases without bound then p_R^* becomes zero, which is not practically substantiated. So C needs to be finite. Additionally, we can assume that at each level of the hierarchy, we have $C_R \propto E \Rightarrow C_R = \rho_R \cdot E$ following the conclusions from Becker (1968), and also demonstrating why there might be a need to differentiate between C_R and C_G. To get an intuitive sense of what ρ_R might be, we can consider it to be the severity of the punishment for Bribe Receivers, who might be punished more harshly under some judicial systems than Bribe Givers. For a quick justification, intuitively it is pretty clear that at the equilibrium $E\times\ (1-\ t) - C_R\times\ t = 0$, where t is the probability representing the perceived threat of being caught. Simplifying, we have $C_R = \frac{1-k}{k} \cdot E$. If we consider $\frac{1-k}{k} = \rho$, we have the simple proportionality relationship, $C_R = \rho_R \cdot E$.

Now, there is an important point to note here. We are considering q to be an exogenous variable, which is odd, as it can be considered to be the indicator of the prevalence of bribery in this network, which is what we are targeting to reduce, so, in fact, it should be affected by the strategy and not exogenous. However, while that is true, it should also be noted that we are simply talking about one period here where q indeed is fixed and exogenous, if we describe multiple periods of this game, however, that is no longer going to be true. Moreover, even more importantly, we are decomposing $\mathbb{P}$ (Bribe | No Sting) into q and β, which is a binary indicator function that shows if the current value of p_G in the given level is sufficient to stop players from the immediately lower level from giving bribes (then $\beta_G = 1$) or if it is not ($\beta_G = 0$). So, we have two different ways that a player may not get a bribe even if we assume that their subordinate is not a sting operator, (i) their subordinate is a "non-bribe" type, or (ii) the p_G is high enough to have deterred their subordinate into not offering a bribe.

Let's examine this further. As can be inferred, if $\beta_G = 0$, then $p = 0$, as $k = 1$, which means that if a bribe is offered it must be a sting offer, and as a result, the necessary p for deterring acceptance of bribes is 0. What does this imply? This seems counter-intuitive, as the expression for p_R seems to contain p itself, but we have to keep in mind that we are dealing with two different p here. Firstly, we need to note, that for any given layer, p_R, which indicates the number of operators necessary to stop players from receiving bribes, indicates the number of operators in the immediately lower layer, as they are the ones giving the bribes. So, p_R is not a characteristic of the layer we are analyzing itself, but that of the next layer. Secondly, p_G, which indicates the number of operators placed to deter bribe-giving, is a characteristic of the layer itself, as the lower layer gives bribes to the layer of interest. So, the expression for p_R simply implies that as long as our $p_G \geq p_G^*$, (i.e. the bribe giving is deterred), then p_R can be 0. This illustrates an important relationship between bribe-giving and receiving (which are just the two sides of the same transaction) as well as between the placement of agents in two adjacent layers.

Next, we look at the Bribe-Giving Game from the viewpoint of a single player for a single time period, understanding the characteristics of p_G better, which has a significant role in our Receiving Game that we just examined:

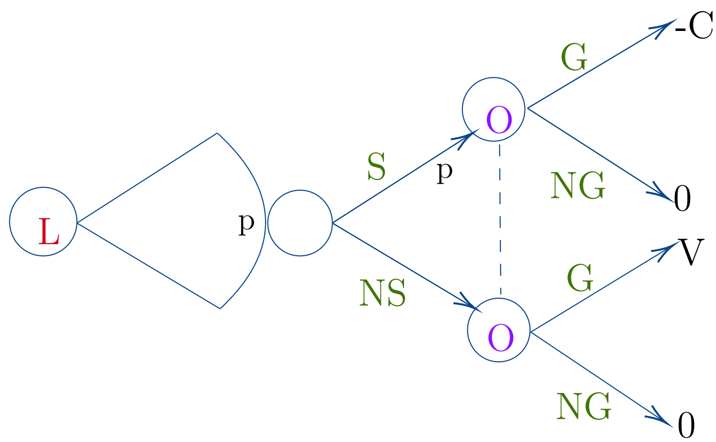

Figure 6. A Closer Look at the Giving Game

As we are looking at the game from the viewpoint of a single player, it is okay to fix the amount of advantage gained from giving a bribe, $V - E$, and we can dispense with the i notation for the other variables as well again. We know that the expected payoff for giving a bribe to any player is:

Payoff (Giving Bribe) = (V - E) $\times\ \mathbb{P}\ (p_R^* < p_R) \times (1 - \mathbb{P}$ (Sting)) $-$ C $\times\ \mathbb{P}$ (Sting) $= 0$

$$\Rightarrow (\text{V - E}) \times \beta \times (1 - p_G) - \text{C} \times p_G = 0$$

So the p_G that induces the indifference condition is: $p_G^* = \frac{(\text{V - E}) \times \beta}{(\text{V - E}) \times \beta + \text{C}}$

Note here, as it is the player's decision whether to give or not give a bribe, the second move of nature, which was present in the receiving game, is absent here. However, again, p_G is inversely proportional to C here, which raises the same concerns as earlier. We again assume $C_G \propto (V - E) \Rightarrow C_G = \rho_G(V - E)$ just as in the Receiving Game, where ρ_G can be intuited as the severity of punishment for Bribe Givers, which might differ from that of Bribe Receivers in certain cases. Also, similar to the Receiving Game, p_G^* is related to the p_R of the above layer and is zero if p_R is high enough. So the key takeaway here is that we only need to hit the lower of the two critical p^* values, p_G^* or p_R^* for a given layer, the other one can be zero.

How then do we connect E to $V - E$? That will allow us to connect the Receiving Game to the Giving Game and draw conclusions that we cannot otherwise. The most intuitive way of doing that is by incorporating a bargaining model into our games. The total value of a bribe transaction is the sum of the value to the receiver added to that of the giver, i.e. $E + (V - E) = V$. So, knowing the aforementioned breakdown of the total value of the transaction, we can model how this benefit might be distributed between the two parties, in other words, how might E relate to $V - E$? (This is our driving question anyway). To tackle this, we can assign weights to the benefit received by each party, depending on their 'bargaining power' to determine the exact distribution. Let π be the bribe Giver's bargaining power, then the naive bargaining model for a single transaction can be stated as

$$\pi \cdot (E) = (1 - \pi) \cdot (V - E)$$

which more generally can be expressed as:

π $\times$ *(Receiver's Benefit - Receiver's Outside Option)* = $(1 - \pi)$ $\times$ *(Giver's Benefit - Giver's Outside Option)*

There are a couple of things to note here. First, the Receiver and Giver's bargaining powers add up to 1. Second, in our case, neither party has an outside option except for refusing to give or take the bribe, in which case they get a payoff of zero. Lastly, it can be argued that the Receiver's and the Giver's payoffs should account for the probabilities of them getting caught, but that complication would make the whole purpose of this model, to have a simple connection between E and $V - E$, meaningless. So that is why we are going to use this current Naive model of bargaining.

Given these assumptions, the task of linking E and $V - E$ is immensely simplified. Simplifying the equation above, we get

$$E = V \cdot (1 - \pi)$$

$$\Rightarrow \quad V - E = \pi \cdot V = \frac{\pi}{1 - \pi} \cdot E$$

Then, if we consider that the bargaining power of the Receiver and the Giver are equal, i.e. $\pi = 0.5$, we have $E = V - E$. Otherwise, we can also assume that $\pi < 0.5$, which might be realistically more true as the higher-up official who is supposed to receive the bribe is likely to have more bargaining power. Regardless, different values of π can lead to different conclusions. One interesting thing to note here is the notion that π could depend on the *degree* (which will be defined and discussed in depth later) of the Receiver. However, without scarcity (which in this context will mean that the supervisor only accepts one bribe, and therefore the subordinates have to compete with each other to bribe them), that model is not fully justifiable.

4. Complete Deterrence

In this section, we begin our preliminary analysis by answering the following question.

Question 4.1. *What is the smallest cost for which you can prevent all corruption in a hierarchy?*

4.1. Solving a Three-Level Hierarchy.

How then do we find the optimal p^* for each layer to deter all corruption? The primary simplicity of analyzing the deterrence of all corruption within the hierarchy comes from the fact that the benefits are the same no matter what method we use. This makes comparisons much easier. To better understand what viable strategies might exist, we will be fully analyzing a three-layered hierarchy example. Imagine we have the following three-layered hierarchy:

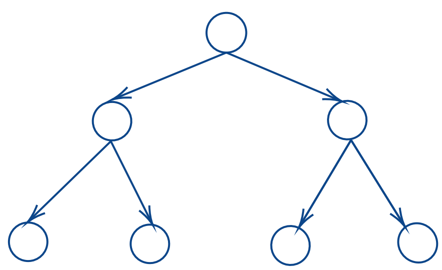

Figure 7. Three Layered Hierarchy

Proposition 4.2. *The Optimal Level(s) to place sting operators vary by the values of the exogenous variables in our cost functions, ρ_R, $q \cdot \rho_G$, and α:*

i. *If $q \cdot \rho_G < \rho_R$, but $q \cdot \rho_G \approx \rho_R$, then under the assumption that the hierarchy is a top-heavy cost structure, we get that placing Sting Operators in only Level 2 is the best strategy. Conversely, if $q \cdot \rho_G > \rho_R$, but $q \cdot \rho_G \approx \rho_R$, bottom-heavy costs imply the same.*
ii. *If $q \cdot \rho_G < \rho_R$, but $q \cdot \rho_G \approx \rho_R$, then under the assumption that the hierarchy is a bottom-heavy cost structure, we get that placing Sting Operators in Levels 1 & 3 is the best strategy. Conversely, if $q \cdot \rho_G > \rho_R$, but $q \cdot \rho_G \approx \rho_R$, top-heavy costs imply the same.*

iii. If $q \cdot \rho_G << \rho_R$, top-heavy costs imply that placing sting agents in Levels 2 & 3 is the best strategy. Conversely, if $q \cdot \rho_G >> \rho_R$, bottom-heavy costs imply that placing sting agents in Levels 1 & 2 is the best strategy.

iv. If $q \cdot \rho_G << \rho_R$, under certain bottom-heavy cost structures, placing sting agents in Levels 1 & 3 is the best strategy. Conversely, if $q \cdot \rho_G >> \rho_R$, under certain top-heavy cost structures the best strategy might be the same.

We will now do a sketch/detailed explanation of Proposition 4.2, examining all the different possibilities under all conditions and drawing our conclusions. Before we list the various options for full deterrence, a few things to note here are: in this case, for each possibility, as we have already discussed, we have that the cost of placing a single agent in a given layer, I_i remains the same, as does the number of players in a given layer, N_i, but the $\mathbb{P}\ (Sting)$ for each layer, p_i can be different for each option, which is why we use a superscript to denote the option for which we are talking about p_i. Additionally, as we mentioned earlier, we are going to consider V, E, and C to be different for each layer, so for each layer, we are going to denote them with a subscript for the layer. Then, to stop all corruption in this hierarchy we have the following options for layers to place agents.

(1) Layer 2: TC $= I_2 \cdot p_2^1 \cdot N_2$

(2) Layer 1 & Layer 2: $TC = I_1 \cdot p_1^2 \cdot N_1 + I_2 \cdot p_2^2 \cdot N_2$

(3) Layer 2 & Layer 3: $TC = I_2 \cdot p_2^3 \cdot N_2 + I_3 \cdot p_3^3 \cdot N_3$

(4) Layer 1 & Layer 3: $TC = I_1 \cdot p_1^4 \cdot N_1 + I_3 \cdot p_3^4 \cdot N_3$

(5) Layer 1 & Layer 2 & Layer 3: $TC = I_1 \cdot p_1^5 \cdot N_1 + I_2 \cdot p_2^5 \cdot N_2 + I_3 \cdot p_3^5 \cdot N_3$

Importantly, as the magnitude of the penalty is proportional to the gains from the bribe transaction, $C_R = \rho_R \cdot E$, and $C_G = \rho_G \cdot (V - E)$ for each level. Another thing to note here is that there are two more possible options where we can place sting operators, which are only in Layer 1 or only in Layer 3. However, these cannot stop all the corruption in the hierarchy, so we do not consider them here.

To begin our analysis, we take a closer look at the p_i^j, and the resulting Total Costs from each of our choices:

(1) Here, p_2^1 has two roles. It is supposed to stop all bribe-giving from Layer 3 and all bribe-receiving from Layer 1. For the first case, we need $p_2^1 \geq \frac{V_2-E_2}{V_2-E_2+C_3}$, and for the second case, we need $p_2^1 \geq \frac{q\cdot E_1}{q\cdot E_1+C_1}$ (as there are no other sting operators we can safely assume $\beta = 1$ in both cases), respectively. So overall, the cutoff for $p_2^1 = \max\{\frac{V_2-E_2}{V_2-E_2+C_3}, \frac{q\cdot E_1}{q\cdot E_1+C_1}\}$. So the resulting Total Cost is $(I_2 \cdot N_2)\cdot\max\{\frac{V_2-E_2}{V_2-E_2+C_3}, \frac{q\cdot E_1}{q\cdot E_1+C_1}\} = \alpha\cdot(I_3 \cdot N_3)\cdot\max\{\frac{V_2-E_2}{V_2-E_2+C_3}, \frac{q\cdot E_1}{q\cdot E_1+C_1}\}$.

(2) Here, the roles of p_1^2 and p_2^2 are to exclusively stop Layer 2 and Layer 3 from giving bribes respectively. So, the cutoffs are $p_1^2 = \frac{V_{1,2}-E_1}{V_1-E_1+C_2}$ and $p_2^2 = \frac{V_2-E_2}{V_2-E_2+C_3}$. Here it could be argued that β does not have to be zero, but as we are not examining the probabilities necessary to stop receiving on any layer, and focusing on bribe giving on both the layers, we have that $\beta = 1$. So the resulting Total Cost is $(I_1 \cdot N_1) \cdot \frac{V_1-E_1}{V_1-E_1+C_2} + (I_2 \cdot N_2) \cdot \frac{V_2-E_2}{V_2-E_2+C_3} = \alpha^2 \cdot (I_3 \cdot N_3) \cdot \frac{V_1-E_1}{V_1-E_1+C_2} + \alpha \cdot (I_3 \cdot N_3) \cdot \frac{V_2-E_2}{V_2-E_2+C_3}$.

(3) Here, the roles of p_2^3 and p_3^3 are to exclusively stop Layer 1 and Layer 2 from receiving bribes respectively. So, the cutoffs are $p_2^3 = \frac{q\cdot E_1}{q\cdot E_1+C_1}$ and $p_3^3 = \frac{q\cdot E_2}{q\cdot E_2+C_2}$. Again, for the same reasons as the above option, we have $\beta = 1$ here. So the resulting Total Cost is $(I_2 \cdot N_2) \cdot \frac{q\cdot E_1}{q\cdot E_1+C_1} + (I_3 \cdot N_3) \cdot \frac{q\cdot E_2}{q\cdot E_2+C_2} = \alpha \cdot (I_3 \cdot N_3) \cdot \frac{q\cdot E_1}{q\cdot E_1+C_1} + (I_3 \cdot N_3) \cdot \frac{q\cdot E_2}{q\cdot E_2+C_2}$

(4) Here, the roles of p_1^4 and p_3^4 are to exclusively stop Layer 2 from giving bribes and Layer 2 from receiving bribes respectively. So, the cutoffs are $p_1^4 = \frac{V_1-E_1}{V_1-E_1+C_2}$ and $p_3^4 = \frac{q\cdot E_2}{q\cdot E_2+C_2}$. $\beta = 1$ here as there are no sting operators in Layer 2. So the resulting Total Cost is $(I_1 \cdot N_1) \cdot \frac{V_1-E_1}{V_1-E_1+C_2} + (I_3 \cdot N_3) \cdot \frac{q\cdot E_2}{q\cdot E_2+C_2} = \alpha^2 \cdot (I_3 \cdot N_3) \cdot \frac{V_1-E_1}{V_1-E_1+C_2} + (I_3 \cdot N_3) \cdot \frac{q\cdot E_2}{q\cdot E_2+C_2}$.

(5) Here, the roles of p_1^5 and p_3^5 are exactly the same as above, which means that the Layer 2 operators are redundant (you can also say the same about Layer 1 and Layer 3 operators if we consider the other Layer-pair sting operators to be the useful ones). So the cutoffs are $p_1^5 = \frac{V_1-E_1}{V_1-E_1+C_2}$, $p_3^5 = \frac{q\cdot E_2}{q\cdot E_2+C_2}$, and $p_2^5 = \min\{\frac{V_2-E_2}{V_2-E_2+C_3}, \frac{q\cdot E_1}{q\cdot E_1+C_1}\}$. If we consider $\beta = 0$ in either case then we have a situation identical to either Option 2 or Option 3. Here, I am not going to elaborate on the Total Costs.

What then is the most efficient strategy for law enforcement? We have already assumed that the benefits from receiving/giving bribes and the penalty of getting caught for each layer are scaled by the same factor, so $\frac{V_1-E_1}{V_1-E_1+C_2} = \frac{V_2-E_2}{V_2-E_2+C_3} = \frac{1}{1+\rho_G}$ and $\frac{q \cdot E_2}{q \cdot E_2+C_2} = \frac{q \cdot E_1}{q \cdot E_1+C_1} = \frac{q}{q+\rho_R}$. So, if we consider $\frac{V_1-E_1}{V_1-E_1+C_2} > \frac{q \cdot E_2}{q \cdot E_2+C_2}$, then by assumption, $\frac{V_2-E_2}{V_2-E_2+C_3} > \frac{q \cdot E_1}{q \cdot E_1+C_1}$ as well, which makes our task much simpler. Then simplifying the Total Costs for each of the options, we have:

(1) $\alpha \cdot \max\{\frac{1}{1+\rho_G}, \frac{q}{q+\rho_R}\}$
(2) $(\alpha^2+\alpha) \cdot \frac{1}{1+\rho_G}$
(3) $(\alpha+1) \cdot \frac{q}{q+\rho_R}$
(4) $\frac{q}{q+\rho_R} + \alpha^2 \cdot \frac{1}{1+\rho_G}$

One of the interesting points here is that the costs are independent of π, which means that the bargaining power has no role to play here. This happens as the size of the penalties takes into account the player's bargaining power, as they are dependent on how much they gain from the transaction. It is also clear that regardless of which of $\frac{V-E}{(V-E)+C} = \frac{1}{1+\rho_G}$ or $\frac{q \cdot E}{q \cdot E+C} = \frac{q}{q+\rho_R}$ are greater, Option (5) is the most inefficient compared to everything else. Now, let us consider the two cases carefully.

- If $\frac{\rho_R}{\rho_G} > q$, then $\frac{V-E}{(V-E)+C} > \frac{q \cdot E}{q \cdot E+C} \Rightarrow \frac{1}{1+\rho_G} > \frac{q}{q+\rho_R}$, and Option (1) is always more efficient than Option (2). This makes sense, as we want to use the option that utilizes the lower value of p more to conserve costs. So the question we need to answer is, which of Options (1), (3), or (4) is more efficient? After calculations, we have Option (1) is more efficient than (4) if $\alpha^2 - \alpha > -q \cdot \frac{1+\rho_G}{q+\rho_R}$; Option (3) is more efficient than (1) if $\alpha > q \cdot \frac{1+\rho_G}{\rho_R - q \cdot \rho_G}$ (note that under our current assumptions $\rho_R - q \cdot \rho_G > 0$); Finally, Option (4) is more efficient than (3) if $\alpha < q \cdot \frac{1+\rho_G}{q+\rho_R}$ (it can be demonstrated that under the current assumptions $q \cdot \frac{1+\rho_G}{q+\rho_R} > \rho_G$, so $\alpha > \rho_G$ is a sufficient condition for Option (4) to be more efficient than (3)).

  Comparing, we have that Option (1) is optimal if $q \cdot \frac{1+\rho_G}{\rho_R - q \cdot \rho_G} > \alpha > q \cdot \frac{1+\rho_G}{q+\rho_R}$. Option (3) is optimal if $\alpha > q \cdot \frac{1+\rho_G}{\rho_R - q \cdot \rho_G}$. Option (4) is optimal if $\alpha < q \cdot \frac{1+\rho_G}{q+\rho_R}$.

  What can we decipher from these conditions? Firstly, the closer together in value ρ_R and $q \cdot \rho_G$ are, the greater the range of possible values that α can take for Option (1) to be optimal. Secondly, the largest α for which Option (4) is optimal is 1, when $\rho_R = q \cdot \rho_G$; the higher ρ_R is compared to ρ_G the smaller the range is for (4) to be optimal. Lastly, the smallest α for which Option (3) is optimal is q, when $\rho_R \to 1$ and $\rho_G \to 0$; the closer ρ_R is compared to ρ_G the smaller the range is for (3) to possibly be optimal.

How do we intuitively interpret/explain these results? The optimal strategy seems to depend on the relationship between ρ_R and ρ_G as well as the value of α. Firstly, in this case, we know that if $\alpha > 1$, i.e. the cost structure is top-heavy, then Option (1) is cheaper than (4). Secondly, the greater ρ_R is compared to $q \cdot \rho_G$, the more heavily bribe-receiving is punished is compared to bribe-giving (meaning that bribe-receivers are already warier than bribe-givers, and thus they can be deterred more easily), Option (3), which depends on deterring bribe-receiving the most, has a greater range of values α where it is optimal. However, if that is not the case, which means that bribe-givers and receivers are punished with relatively comparable severity, we have that Option (1) is still the best. Of course, if the costs are still too bottom-heavy i.e. $\alpha << 1$, then that can offset even a high disparity between the punishments for bribe-giving and receiving making Option (4) optimal, as for (4) we place agents in Level 1, which is going to be by far the cheapest. Finally, as a converse to the last point, if the severity of punishment for bribe-giver and receivers are very close to each other, a bottom-heavy cost structure means that placing agents in Level 1 and Level 3 is cheaper than just Level 2, simply by construction of α, leading to Option (4) being optimal.

- If the situation was flipped and $\frac{\rho_R}{\rho_G} < q$, then $\frac{V-E}{(V-E)+C} < \frac{q \cdot E}{q \cdot E + C} \Rightarrow \frac{1}{1+\rho_G} < \frac{q}{q+\rho_R}$, and Option (1) is always more efficient than Option (3). Again, this makes sense as under this condition, placing sting operators in Level 3 is redundant. So the question we need to answer is, which of Options (1), (2), or (4) is more efficient? After calculations, we have Option (1) is more efficient than (4) if $\frac{\alpha^2}{\alpha - 1} > q \cdot \frac{1+\rho_G}{q+\rho_R}$; Option (2) is more efficient than (1) if $\alpha < \frac{q \cdot \rho_G - \rho_R}{q + \rho_R}$ (note that under our current assumptions $q \cdot \rho_G - \rho_R > 0$); Finally, Option (4) is more efficient than (2) if $\alpha > q \cdot \frac{1+\rho_G}{q+\rho_R}$ (it can be demonstrated that under the current assumptions $q \cdot \frac{1+\rho_G}{q+\rho_R} > \rho_G$, so $\alpha > \rho_G$ is a necessary condition for Option (4) to be more efficient than (2)).

Comparing, we have that Option (1) is optimal if $q \cdot \frac{q \cdot \rho_G - \rho_R}{q + \rho_R} < \alpha < q \cdot \frac{1+\rho_G}{q+\rho_R}$. Option (2) is optimal if $\alpha > q \cdot \frac{1+\rho_G}{\rho_R - q \cdot \rho_G}$. Option (4) is optimal if $\alpha < q \cdot \frac{1+\rho_G}{q+\rho_R}$.

What can we decipher from these conditions? Firstly, when ρ_R and $q \cdot \rho_G$ are very close together in value, Option (1) is optimal if $0 < \alpha < 1$, i.e. the cost is bottom-heavy. Conversely, if $\rho_R \to 1$ and $\rho_G \to 0$, then Option (1) is optimal if $1 < \alpha < 2$. Secondly, the lowest α for which Option (4) can be optimal is 1, when ρ_R is very close in value to $\cdot \rho_G$. So, if the costs are top-heavy, depending on the relationship between ρ_R and ρ_G, we will have either (1) or (4) as optimal. Lastly, the highest α for which Option (2) is optimal is 1 when $\rho_R \to 1$ and $\rho_G \to 0$; the closer ρ_R is compared to ρ_G the smaller the range is for (2) to be possibly optimal. So, if the costs are bottom-heavy, depending on the relationship between ρ_R and ρ_G, we will have either (1) or (2) as optimal.

How do we intuitively interpret/explain these results? Firstly, in this case, we know that if $\alpha < 1$, i.e. the cost structure is bottom-heavy, then Option (1) is cheaper than (4). Secondly, the more heavily bribe-giving is punished compared to bribe-receiving, Option (2), which depends on deterring bribe-giving the most, has a greater range of values α where it is optimal. However, if that is not the case, which means that bribe-givers and receivers are punished with relatively comparable severity, we have that Option (1) is still the best. Of course, if the costs are still too top-heavy i.e. $\alpha >> 1$, then that can offset even a high disparity between the punishments for bribe-giving and receiving making Option (4) optimal, as for (4) we place agents in Level 3, which is going to be by far the cheapest. Finally, as a converse to the last point, if the severity of punishment for bribe-giver and receivers are very close to each other, a top-heavy cost structure means that placing agents in Level 1 and Level 3 is cheaper than just Level 2, simply by construction of α, leading to Option (4) being optimal.

Question 4.3. *Which of the situations elaborated above is practically most likely to occur?*

To answer this we need to discuss the empirical side of the question of corruption to see what is most commonplace in countries around the world. In many countries such as India, the United States, the UK, and France, bribe-givers and bribe-receivers are punished equally. On the other hand, in countries like China, Japan, and Russia, the bribe-receivers are punished more severely than the bribe-givers‘. There has been quite some research into the effectiveness of asymmetric punishments and there are some who conclude that punishing bribe-receivers more harshly is more efficient, while others maintain that symmetric punishments are most effective (Basu, Basu, & Cordella, 2014). To us, however, this is a purely erogenous variable, so we will not be particularly engaging with this research except to just look at the data available.

There is one subtle point to make here, however. The relative values of ρ_G and ρ_R do not directly reflect the relative severity of the C^G and C^R, primarily because they depend on π, which in some sense already causes the punishment to be asymmetric if $\pi \neq 0.5$. So, in a truer sense, we have that ρ_G and ρ_R capture the difference in severity, taking into account the bargaining power of each party. Generally, however, it will still be true that the relative values of ρ_G and ρ_R will reflect what we would expect, possibly in a more balanced manner.

With this in mind, however, we can say that generally in the world, and in India, from where we draw our inspiration, the law seems to be against bribe-receivers more, but the difference in severity is not high. Now the only question left to answer is, do we expect the costs to be top or bottom-heavy? In my opinion, generally, due to the fact that the costs of placing an agent very close to the top of the hierarchy might just be prohibitively high, simply due to the qualifications needed to do so, costs are usually top-heavy. With these assumptions, we know that Option (1) is the best option.

4.2. **Layered-Cake Conjecture.**

We have fully analyzed a three-layered hierarchy. What about a hierarchy with more levels though? What can we conclude more generally? We now have a good hypothesis for what might be a generally efficient strategy for a hierarchy with any given number of layers, if our objective is to stop all bribery in the network. To achieve that we propose this intuitive solution, which we call the "Layered-Cake" strategy. Despite the fancy name it essentially means that we put sting operators in alternate layers. That way, we at the same time prevent the bribe-giving on the lower layer and the bribe-accepting of the higher layer. So we do not need operators on adjacent layers. We give an example in Figure 8.

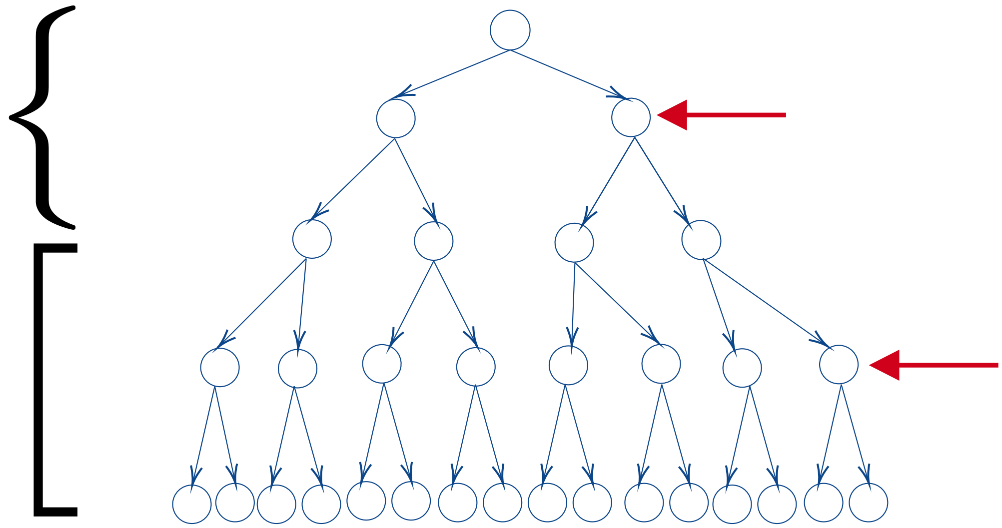

FIGURE 8. A Layered-Cake Strategy

This appears to be essentially an extension of Option (1) to hierarchies with more levels (at least odd-numbered ones). However, how do we know that this generalizes? Examine the 5-Level Hierarchy pictured above. It is technically possible to consider it as two different 3-Level hierarchies, as denoted by the brace and the bracket, with them sharing Layer 3. Now the question is can we minimize the cost of the whole hierarchy by individually minimizing the costs of each smaller one? What about the shared Layer 3? Fortunately, our scope here is limited to examining the generalizability of Option (1), so these problems are not so complicated. If Option (1) is indeed optimal in either of these hierarchies, it must be so in the other as well. This is because the costs for each layer of the { hierarchy is simply a factor of α^2 higher than the cost of the corresponding layer in the [hierarchy, implying that the comparative cost structures are identical. Now, using Option (1) on the { hierarchy would mean placing agents in Level 2 in order to stop the Level 1 players from accepting bribes and Level 3 players from giving bribes. Similarly, using Option (1) on the [hierarchy would mean placing agents in Level 4 in order to stop the Level 3 players from accepting bribes and Level 5 players from giving bribes. Note here that there is no overlap in role between the two. The only possible point of overlap is that both these strategies affect Level 3, but one prevents players there from accepting bribes from the level below, while the other discourages them from giving bribes to the level above, which are events that are completely independent of one another. Therefore, in this case, if for either of the smaller sub-hierarchies Option (1) is optimal it must be for the other as well, and placing agents in Levels 2 and 4 is optimal for the overall larger hierarchy (which happens due to the costs being independent, meaning that we can minimize the overall cost by minimizing each component). Note that this generalization is not so straightforward for the other options, mainly because there is some overlap while placing sting agents in the two sub-hierarchies.

However, under which conditions is this the best option? As we have clearly seen from the Three-Level example above, different conditions can certainly influence the optimal strategy. However, for this we already know the answer; they are simply the same conditions for which Option (1) was optimal in the example we solved, as stated in Proposition 4.2. So the same conditions are sufficient for the 5-layered Hierarchy. This solution has been highlighted using arrows. It is important to note here that there is another layer-cake strategy here, which places operators in layers 1, 3, and 5. However, under the current conditions, this is more inefficient, just like Option (4) in our example, as layer 5 and layer 1 operators only prevent either giving or accepting bribes, not both. The same problem happens if there is an even number of layers, as there will be inefficiency there as well. I estimate that of the two layered-cake solutions in an even-layered hierarchy, the preference of one over the other will depend again on the relative values of α and $\frac{q \cdot \rho_G}{\rho_R}$.

5. Deterrence by Layer

After having satisfactorily answered the question of what is the best strategy to achieve complete deterrence, we can explore a slightly more complicated question:

Question 5.1. *How do you best stop corruption at any given level of the hierarchy?*

This question is interesting because, given the costs and benefits of stopping corruption, there might be conditions that suggest we should focus on only one level or a few levels and not the whole hierarchy, as the costs might be prohibitive and the benefits not correspondingly high enough.

Let us start by noting a few key differences between this and the last game. Firstly, here the benefit of each strategy is going to be potentially different, which means that we cannot restrict our analysis to just the costs of the strategies. Secondly, we excluded the strategies of placing sting operators in just Layer 1 or just in Layer 3 in our earlier game, but they are viable strategies here. So we have to consider all possible strategies here. Let's demonstrate some of our possible considerations with an example. Let's again examine our old friend the three-layered hierarchy:

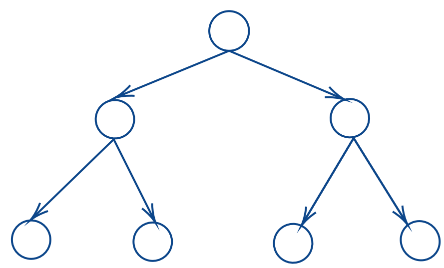

Figure 9. Three Layered Hierarchy

Let us assume we only want to stop the bribery in Layer-pair 2,3. What is the best way of achieving this? To stop bribe-receiving from Layer 2, we need $p_3 = \frac{q \cdot E}{q \cdot E + C} \Rightarrow TC = \frac{q}{q + \rho_R}$. On the other hand, to stop bribe-giving to Layer 2, we need $p_2 = \frac{V - E}{(V-E) + C} \Rightarrow TC = \alpha \cdot \frac{1}{1 + \rho_G}$. So as the benefits of both strategies are exactly the same, we can simply see which of these is greater. Again, if you wanted to stop corruption only in Layer-pair 1,2, you would have the exact same considerations. However, how do you compare the benefits of stopping corruption across the two Layer-pairs? While detailing the costs and the benefits in the model, we suggested a benefit variable for stopping corruption in Layer-Pair i, j to be $H_{i,j}$, but how do we intuitively quantify that? For example, how do you compare the total payoff of placing sting operators in only Layer 1 versus that of placing operators in Layer 3? This is going to be dependent on the structure of the hierarchy, and even if you fix that this is still going to be complex. Therefore, we are going to leave it out of this project, but it is certainly an interesting question.

6. Next Steps

I have already discussed a few of the next steps planned for this model, but there are quite a few others that are worth consideration. Firstly, there needs to be found a general solution that applies to hierarchies with both even and odd numbers of layers, which is something I have yet to figure out, as the Layered-Cake seems to best apply to odd hierarchies. I am certain that there could be a condition proved to show that either of the Layer-cakes possible in a even-hierarchy work under the right values of ρ_G, ρ_R, and α, but I have not had the time to do that yet. The other large question worth exploring is something that I will be going in to in some detail now.

6.1. **The Question of Degree.**

One of the last things I wanted to do with this project was to not only generalize my results for hierarchies of any shape but also examine how they change as the shape of the hierarchy changes. In particular, in the case of hierarchies, we are interested in the *degree* of the hierarchy. The *degree* of a player may be defined as the number of subordinates they are directly attached to on the layer right below them. Here, we generally assume that every player on the same layer has the same degree, so we can talk about the degrees of layers instead of individual players themselves. A *regular* hierarchy is a hierarchy where all layers have equal degrees, and visually most of our examples have been regular hierarchies.

If we assume the geometric cost function that we have in the project so far, where $I_i \cdot N_i = \alpha \cdot I_{i+1} \cdot N_{i+1}$, this question becomes particularly uninteresting as there is no particular implication that degree has on our results. This is because we are essentially assuming that no matter how we change the structure of a certain level, the costs of that level are going to be exactly the same compared to its adjacent levels. However, if we relax this assumption, we can project that we will have more interesting results. For example, on the other extreme, assume that placing any operator in a layer costs the exact same amount regardless of the number of players in the layer. Then, if the degree for a particular layer increases, which means that each player there has more subordinates now, then it becomes relatively more expensive to fulfill the required p in the layer below, making deterring the accepting of bribes harder, and leading to the targeting of higher-up layers as that becomes more lucrative. However, if the degree of all layers changes by the same factor, then the relative cost of placing agents does not change between layers but the absolute cost still might, so it might lead to the law agency not sending in any operators at all.

Something to keep in mind here is that we just assumed that the cost of placing operators in a layer is largely independent of the number of players in the layer, while in the past we have assumed that they are inversely proportionate. In reality, it is probably somewhere in the middle. This means that some features of both of these results translate to reality, even though it remains to be seen which ones they are.

7. Conclusion

This model was a theoretical deep dive into a very specific network structure (regular hierarchies), but with the current notation setup, it certainly has greater scope. For example, incorporating a scarcity model will certainly help us make more sense of the degree question, as we can change the bargaining weights according to that. Furthermore, if we consider the severity of punishment to be a choice variable, we can solve for a full Becker equilibrium, and solve both sides of the story. However, that is beyond the scope of this project, which has still been somewhat insightful about the behavior of the best response for external law agencies given the constraints of cost structures and the law.

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Appendix A. Workings For Proposition 4.2

As promised in Section 4, I wanted to show how we arrived at the bounds we did for Option (1), (2), (3), or (4).

Let us first assume that $\frac{\rho_R}{\rho_G} > q$, then $\frac{V-E}{(V-E)+C} > \frac{q \cdot E}{q \cdot E + C} \Rightarrow \frac{1}{1+\rho_G} > \frac{q}{q+\rho_R}$.

i. Option (1), which has cost $\alpha \cdot \frac{1}{1+\rho_G}$ is always more efficient than Option (2) which has cost $(\alpha^2 + \alpha) \cdot \frac{1}{1+\rho_G}$, so we don't bother with Option (2) at all.

ii. Option (3), which has cost $(\alpha+1) \cdot \frac{q}{q+\rho_R}$, is better than Option (1) if (1) costs greater than (3), i.e.

$$\begin{aligned} & \alpha \cdot \frac{1}{1+\rho_G} - (\alpha+1) \cdot \frac{q}{q+\rho_R} > 0 \\ \Rightarrow\quad & \alpha \cdot (\frac{1}{1+\rho_G} - \frac{q}{q+\rho_R}) > \frac{q}{q+\rho_R} \\ \Rightarrow\quad & \alpha \cdot \frac{q+\rho_R - q - q\rho_G}{(1+\rho_G)(q+\rho_R)} > \frac{q}{q+\rho_R} \\ \Rightarrow\quad & \alpha \cdot \frac{\rho_R - q\rho_G}{(1+\rho_G)} > q \\ \Rightarrow\quad & \alpha > \frac{q \cdot (1+\rho_G)}{\rho_R - q\rho_G} \end{aligned}$$

iii. Option (4), which has cost $\frac{q}{q+\rho_R} + \alpha^2 \cdot \frac{1}{1+\rho_G}$, is better than Option (3) if (3) costs more than (4), i.e.

$$\begin{aligned} & (\alpha+1) \cdot \frac{q}{q+\rho_R} - \frac{q}{q+\rho_R} - \alpha^2 \cdot \frac{1}{1+\rho_G} > 0 \\ \Rightarrow\quad & \alpha \cdot (\frac{q}{q+\rho_R} - \alpha \cdot \frac{1}{1+\rho_G}) > 0 \\ \Rightarrow\quad & \frac{q}{q+\rho_R} > \alpha \cdot \frac{1}{1+\rho_G} \\ \Rightarrow\quad & \alpha < \frac{q \cdot (1+\rho_G)}{q+\rho_R} \end{aligned}$$

iv. The comparison between Option (4) and Option (1) is not useful and more complex, so we can skip that.

Now let us assume that $\frac{\rho_R}{\rho_G} < q$, then $\frac{V-E}{(V-E)+C} < \frac{q \cdot E}{q \cdot E + C} \Rightarrow \frac{1}{1+\rho_G} < \frac{q}{q+\rho_R}$.

i. Option (1), which has cost $\alpha \cdot \frac{q}{q+\rho_R}$ is always more efficient than Option (3) which has cost $(\alpha+1) \cdot \frac{q}{q+\rho_R}$, so we don't bother with Option (3) at all.

ii. Option (2), which has cost $(\alpha^2+\alpha) \cdot \frac{1}{1+\rho_G}$, is better than Option (1) if (1) costs greater than (2), i.e.

$$\begin{aligned} & \alpha \cdot \frac{q}{q+\rho_R} - (\alpha^2+\alpha) \cdot \frac{1}{1+\rho_G} > 0 \\ \Rightarrow \quad & \frac{q}{q+\rho_R} - \frac{1}{1+\rho_G} > \alpha \cdot \frac{1}{1+\rho_G} \\ \Rightarrow \quad & \frac{q + q\rho_G - q - \rho_R}{(1+\rho_G)(q+\rho_R)} > \alpha \cdot \frac{1}{1+\rho_G} \\ \Rightarrow \quad & \frac{q\rho_G - \rho_R}{q+\rho_R} > \alpha \end{aligned}$$

iii. Option (4), which has cost $\frac{q}{q+\rho_R} + \alpha^2 \cdot \frac{1}{1+\rho_G}$, is better than Option (2) if (2) costs more than (4), i.e.

$$\begin{aligned} & (\alpha^2+\alpha) \cdot \frac{1}{1+\rho_G} - \frac{q}{q+\rho_R} - \alpha^2 \cdot \frac{1}{1+\rho_G} > 0 \\ \Rightarrow \quad & \alpha \cdot \frac{1}{1+\rho_G} - \frac{q}{q+\rho_R} > 0 \\ \Rightarrow \quad & \alpha \cdot \frac{1}{1+\rho_G} > \frac{q}{q+\rho_R} \\ \Rightarrow \quad & \alpha > \frac{q \cdot (1+\rho_G)}{q+\rho_R} \end{aligned}$$

iv. The comparison between Option (4) and Option (1) is not useful and more complex, so we can skip that.

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