Composite vortex patterns formed by component light beams with non-integral topological charge

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ABSTRACT

We present a study of composite vortices in light beams using component beams with no integral topological charge. We observed the same general features that are seen in when the component beams have an integral topological charge [E.J. Galvez, N. Smiley, and N. Fernandes, "Composite optical vortices formed by collinear Laguerre-Gauss beams," *Proc. SPIE* **6131**, pp. 19–26, 2006.]. These are: (1) that new vortices appear at distances from the beam that depend on the ratio of the intensity of the component beams, and (2) that the angular location of the vortices depends on the phase difference between them. We also observed that some of the vortices associated with fractional charge that did not follow the same dynamics.

Keywords: Singular Optics, Optical Vortices

1. INTRODUCTION

The wave patterns that result when collinear light beams bearing optical vortices are superimposed constitute an interesting problem in singular optics. These patterns are far from intuitive. They exhibit an abundance of optical vortices whose position is well determined by the general characteristics of the component beams: the individual topological charges, their intensity ratio and relative phase.[1] In this work we extend our previous studies to the case where the component beams have a non-integral topological charge.

Beams with a non-integral topological charge are composite beams themselves because they can be expressed in terms of an infinite superposition of modes of integral topological charge.[3] The general properties of these optical beams have been studied before.[3–6] The relative positions of the vortices that they contain are determined by the relative complex amplitude of the terms in the superposition. Thus, the question of whether the features of composite vortex beams generated with non-integral charge are any different than those with integral-charge component beams begs for an answer.

In this work we investigate this question from an experimental perspective. Because beams with half-integral topological charges are not expressed in a closed form,[3,5] the composite pattern cannot be determined by the simple model used previously.[1] Therefore, our study is only experimental. In Sec. 2 we describe the method that we used in our experiments. In Sec. 3 we summarize previous results of composite beams with components of integral topological charge and the characteristics of beams with non-integral topological charge. Our results are presented in Sec. 4, but divided into three subsections: one for the study of the patterns as a function of the relative intensity of the component beams, another one for the study as a function of the relative phase, and a third subsection to investigate the fate of the fine structure of singularities in half-integer components. We close in Sec. 5 with our conclusions.

2. APPARATUS

A schematic of the setup that we used is shown in Fig. 1. A linearly polarized Gaussian beam from a HeNe laser was sent to two nested Mach-Zehnder interferometers. The light going through the lower branch of the outer interferometer entered the inner interferometer, which was used for making the composite beam. The light going through the upper branch was used as a reference beam for making fringe patterns. We used these patterns to

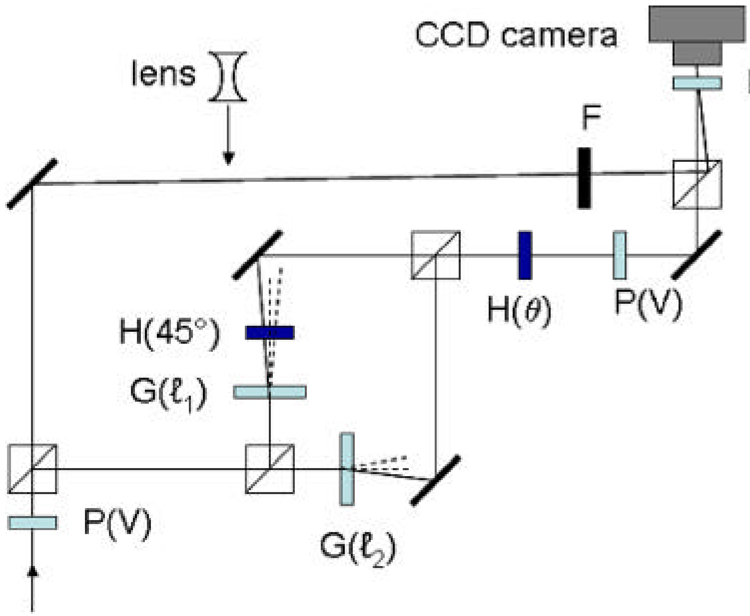

Figure 1. Schematic of the apparatus. It consisted of two nested Mach-Zehnder interferometers with forked diffraction gratings G(ℓ_1) and G(ℓ_2) to make the component beams of charge ℓ_1 and ℓ_2, respectively, and with half-wave plates (H) and polarizers (P) for controlling the relative intensity of the component beams. Neutral density filters (F) and a lens were used to attenuate and expand the reference beam.

identify the optical vortices and dislocations. We typically adjusted the size and intensity of the reference beam with a lens or a beam expander and neutral density filters to obtain the desired interferograms.

The components of the composite beam were generated in the arms of the inner Mach-Zehnder interferometer via forked diffraction gratings (G) of charge ℓ_1 and ℓ_2 located in each of the arms. These were passive binary gratings made by photoreduction of computer-generated patterns.[6] The mirrors in conjunction with the second beam-splitter of the interferometer were adjusted to align one of the first-order diffracted beams of each grating into collinear paths outside the interferometer. All the components were mounted on pedestal mounts for greatest stability against vibrations.

The laser was oriented to emit light polarized in the vertical direction. A vertical polarizer placed before the interferometer eliminated any unwanted horizontal components. In the upper branch of the inner interferometer we placed a half-wave plate (H) with its axis at 45° from the vertical to flip the polarization to the horizontal direction. As a consequence, the component beams of charge ℓ_1 and ℓ_2 were orthogonally polarized as they exited the interferometer. After the inner interferometer the combination of a half-wave plate followed by a polarizer, with the latter set to the vertical direction, served to project the polarization of both components along the same direction and interfere. By rotating the half-wave plate we controlled the relative intensity of the component beams; i.e., when the half-wave plate's axis formed the angle θ with the vertical direction the relative intensity of the composite beams was $I_1/I_2 = \tan^2 2\theta$.

The composite beam combined with the reference beam at the second beam splitter of the outer interferometer. Together they projected a fringe interference pattern onto a charge-coupled-device (CCD) camera. A polarizer placed before the CCD camera was rotated to adjust the intensity of the light reaching it.

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3. SUMMARY OF PREVIOUS WORK

3.1. Composite Vortices

We will start by summarizing the results of our previous work on composite vortices using component beams with integral topological charge.[1] Let us consider two component beams with topological charges ℓ_1 and ℓ_2, of sign $\sigma_1 = \ell_1/|\ell_1|$ and $\sigma_2 = \ell_2/|\sigma_2|$, with intensities I_1 and I_2 respectively, and a relative phase δ.

1. Case $\ell_1 < \ell_2$:
   - The composite beam has $|\ell_1|$ vortices of charge σ_1 located in the center, surrounded by $|\ell_2 - \ell_1|$ vortices of charge σ_2 located symmetrically along the periphery of the composite pattern and at the same radial distance from the center.
   - The radial position of the peripheral vortices is given by

$$r = \frac{w}{\sqrt{2}} \left(\frac{|\ell_2|!}{|\ell_1|!} \frac{I_1}{I_2} \right)^{\frac{1}{2(|\ell_2|-|\ell_1|)}}, \tag{1}$$

     where w is the waist of the beam. That is, the peripheral vortices move either in or out as we vary the relative intensity of the component beams.
   - The angular positions of the peripheral vortices are given by

$$\phi_n = \frac{\delta + n\pi}{\ell_2 - \ell_1}, \tag{2}$$

     where $n = 1, ..., (2|\ell_2 - \ell_1| - 1)$ is an odd integer. As a result, the peripheral vortices rotate about the center of the beam when the relative phase δ is changed.

2. Case $\ell_1 = -\ell_2$:
   - When $I_1 > I_2$ there is a central vortex of charge ℓ_1 with no peripheral vortices.
   - When $I_1 = I_2$ there is no central vortex, no peripheral vortices, but $2|\ell_1|$ radial shear singularities located at angles given by Eq. 2.

3.2. Non-integer Vortex Beams

A non-integral vortex beam of overall charge ℓ can be generated either by passage through a phase-plate of non-integral topological charge ℓ or via first-order diffraction off a forked diffraction grating with a non-integral dislocation of magnitude ℓ.[3–6] The characteristics of such a beam are summarized below.

- The intensity pattern consists of one or more broken concentric rings.
- When ℓ is not a half-integer the number of vortices inside the ring is ℓ rounded to the nearest integer.
- When ℓ is a half-integer the number of vortices inside the ring is ℓ rounded down to the nearest integer. In addition, the cut on the ring contains a chain of singly-charged vortices of alternating sign, with the closest one to the center of the beam, hereafter called the chain leader, having the same sign as ℓ.
- The orbital angular momentum of the beam varies nonlinearly with ℓ

4. RESULTS

In studying the composite beams as a function of the variables at hand we considered composites with all possible combinations of component beams: ones with one component beam with an integral charge and the other one with a non-integral charge, and others where both components had non-integral charge. We also considered half-integer and non-half-integer component beams.

4.1. Intensity Dependence

Figure 2 shows interferograms of the composite vortices created when one component beam had the charge $\ell_2 = +2.5$. In the top row we show the case when the other beam had a charge $\ell_1 = -1$, and in the bottom row the case when the other beam had a charge $\ell_1 = +1$. The columns correspond to different normalized intensities, as labeled in the figure. We took more interferograms, but the frames shown in the figure are representative of the essential phenomena.

The vortices in the interferogram can be found by locating the forks in the fringe pattern. The direction of the tines of each fork allows us to identify the sign of the vortices. In the case of Fig. 2 forks with tines pointing to the right have a positive charge and conversely, forks with tines pointing to the left have a negative charge. The sign of the charge determines the direction, clockwise or counter-clockwise, in which the phase advances when going on a circular loop around the phase singularity. The two frames on the left-most column correspond to beams with $\ell_1 = -1$ (upper frame) and $\ell_1 = +1$ (lower frame). On the right-most column we observe the patterns of $\ell_2 = +2.5$, which show two "+1" vortices in the central region and an additional "+1" vortex on the left side breaking an otherwise continuous ring, as is typical of a half-integer vortex beam.

In both cases we observed the same consistent pattern, which is similar to the case of integral component beams. Let us first focus on the case of the top row. As we decrease I_1/I_2, going from left to right in the figure, we see three "+1" vortices with roughly equal angular separation enter the composite beam from the periphery. We also observe an additional "+1" vortex appear on the upper left side of the composite beam. However, the latter vortex does not move much once it appears. It remains in the periphery until the beam is made up only of the non-integral component. We interpret this vortex as the one leading the chain of vortices of alternating sign that is typical of half-inter vortex beams.[3–6]

What is interesting about the observed pattern is the different roles or paths followed by the three peripheral and the vortex-chain leader. The three peripheral vortices play the same role as in the case of the composite beam with integral components. That is, they move in and out as a function of the relative intensity. The chain-leader vortex and presumably its accompanying chain (see below) play a different role. They basically appear or disappear, and when present they do not seem move much as the intensity ratio is changed.

The case of the bottom row of Fig. 2 is similar. When looking at the sequence of frames from left to right we see two vortices appear on the left side, one, the peripheral vortex, moves all the way to the center as the intensity ratio is decreased. The other one, the chain leader, moves up to the edge of the pattern and stays there. The chain of alternating vortices is not seen in the figure due to the orientation and resolution of the fringe pattern, but we believe they are there. We study them in more detail in Sec. 4.3.

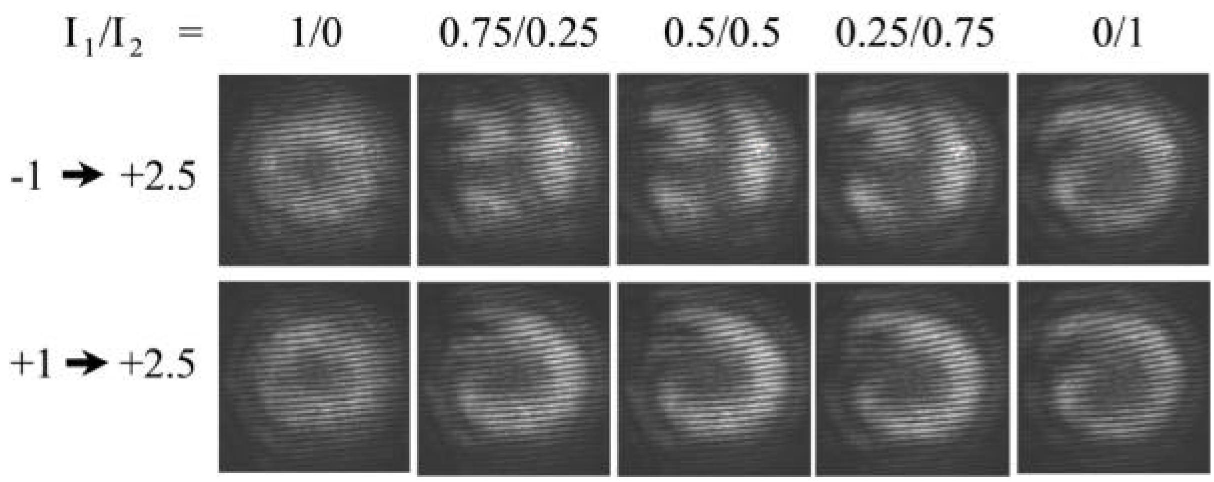

Figure 2. Interferograms for the case when the component beams had charges $\ell_2 = +2.5$ and $\ell_1 = \pm 1$. The normalized intensities of the component beams I_1/I_2 are given on top of each column.

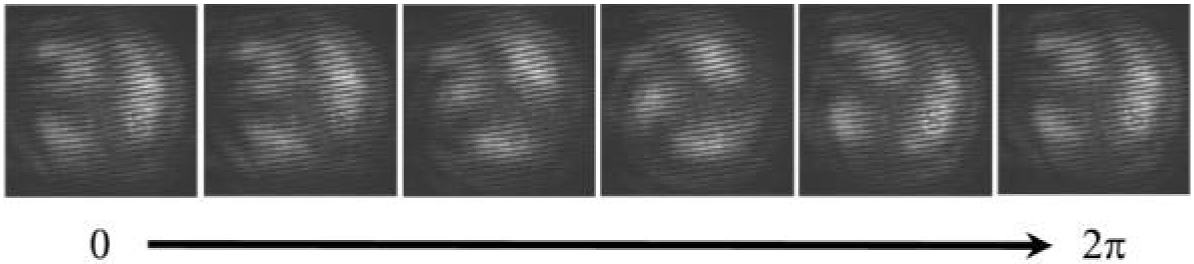

Figure 3. Interferograms for the case when the component beams had charges $\ell_2 = +2.5$ and $\ell_1 = -1$, and when their intensities were the same (i.e., $I_1 = I_2$). The phase between the two component beams was varied from left to right on roughly equal intervals.

In most cases the number of peripherals was consistent with $|\ell_2 - \ell_1|$ rounded to the nearest integer. For example, when $\ell_1 = -2/3$ and $\ell_2 = +8/3$ we saw three peripheral vortices, and when $\ell_1 = +2/3$ and $\ell_2 = +8/3$ there were two. The exceptions depended on the specific values of ℓ_1 and ℓ_2. The case of Fig. 2 was one. Since the center of the beam with $\ell = +2.5$ has two central vortices, only three peripheral vortices are needed to convert the center of the beam from $\ell = -1$ to one with $\ell = +2.5$.

4.2. Phase Dependence

We also studied the dependence of the patterns on the relative phase between the two component beams. We varied the phase by mechanically pulling the mount of one of the mirrors of the interferometer. Alternatively we applied a voltage to a piezo electric ceramic placed as a spacer in a translation stage where one of the interferometer mirrors was mounted.

In the patterns shown in Fig. 3 we adjusted the intensity of the component beams to be the same. As we varied the phase between the component beams we see that the peripheral vortices rotate about the center, as is the case for integral components. However, the vortex-chain leader does not rotate. We do not see a doubly-charged vortex when one of the peripherals comes to the location of the vortex-chain leader. Perhaps the chain of alternating vortices is not strong enough to survive the phase warping effected by a peripheral vortex located in the neighborhood.

The peripheral vortices and vortex-chain leader mark the pattern via dark regions (i.e., the darkness associated with the phase singularities). We could distinguish them by varying the phase δ. As a consequence, the peripheral vortices and their associated dark regions rotated but the vortex-chain leader did not rotate. In some cases the vortex chain region appeared as a region of sheared phase with no clear fork standing out.

In cases where both components had a fractional charge we had to use the rotation of the pattern to help identify the peripheral vortices. These cases were not clear cut. For example, when $\ell_1 = -1.5$ and $\ell_2 = +5/3$ we saw several darkened regions that rotated with forks in all but one of them. It seemed that the darkened region with no forks was a place where an incoming peripheral vortex of one sign canceled a central one of the other sign. In cases where the non-integral part of ℓ_1 was greater than 0.5, such as the case of $\ell_1 = 5/3$ and $\ell_2 = -8/3$, we saw one of the central vortices exit the beam through one of five darkened regions as I_1/I_2 was decreased.

The investigation of beams with $\ell_1 = -\ell_2$ was far more complicated than expected. When $I_1 = I_2$ the beam pattern included regions of shear phase singularity that rotated with δ, as for the case of integral components. The number of these regions depended on ℓ_1. For example, Fig. 4 shows in frame (a) a composite with four darkened regions when $\ell_1 = 1.5$. Conversely, frame (b) shows a composite with five darkened regions when $\ell_1 = 8/3$. The darkened regions moved over by one position when δ increased by 2π. Depending on the values of ℓ_1 and ℓ_2 the darkened regions may have single vortices instead of a phase-shear region, or may not have any vortex at all, as seen in Fig. 4. This is a case where an analytical theory could clarify the observations.

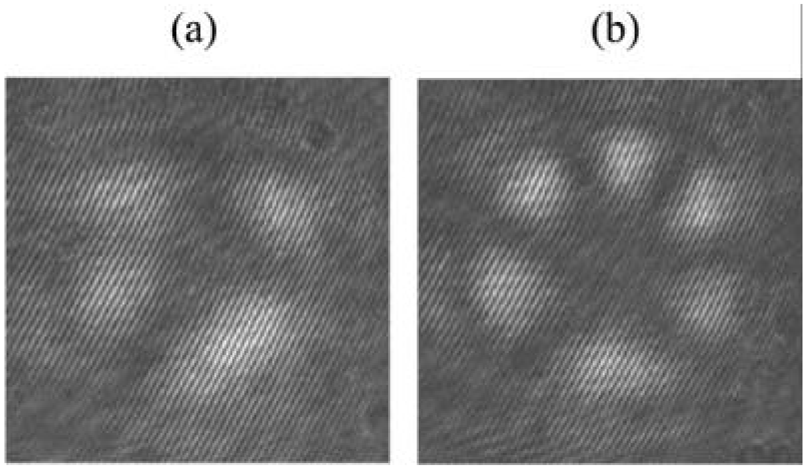

Figure 4. Interferograms for the case when the component beams had a fractional charge of the same magnitude: (a) $\ell_1 = -\ell_2 = 1.5$ and (b) $\ell_1 = -\ell_2 = 8/3$. In both examples $I_1 = I_2$.

4.3. Half-integer Components

The composite beams that are obtained when the two beams are non-integral follow the same general rules as above, although not precisely because the wavefront of the component beams is not uniform as in the case of integral beams. Despite this we see a consistent pattern of peripheral vortices moving in and out of the composite pattern when changing the ratio of intensities.

One question that remains is the fate of the infinite chain of alternating vortices that denote beams with half-integral charge. Figure 5 represents an example of our effort in studying this case. The component beams that we chose had their half-integer "cut" in opposite sides of the beam as seen in frames (a) and (c), which correspond to beams with charge -1.5 and $+2.5$, respectively. The top row shows our raw interferograms. In this case we put a fine fringe spacing to identify the vortex chains. In the bottom row we labeled the vortices that we could identify. The middle frames (b) correspond to the case when the two components had equal intensity.

In the case of Fig. 5 we see that the chains of alternating vortices do not fizzle. The vortices of the chain move around locally when the peripheral vortices are not near them. One can follow the paths of the vortices as the intensity ratio is varied. Thus, we conclude that this case, although complex, continues to follow the general pattern. We observed slightly different patterns of forks for different values of δ. Thus, the exact pattern of chain-vortices is very sensitive to the location of the peripheral vortices. It is surprising to see that the chains of alternating vortices do not totally disappear even though the beam is an even mixture of the two component beams.

5. CONCLUSIONS

In summary, we have performed a study of the vortex patterns that are present in composite beams generated when the two collinear component beams have topological charges that are non-integral. We observed the same general features of the integral case: that peripheral vortices move in and out of the field of the composite pattern as a function of the relative intensity of the component beams, and the rotation of these about the center of the beam as the phase between the two beams is changed. The number of peripheral vortices was consistent with $|\ell_2 - \ell_1|$ rounded to the neared integer, but there were exceptions, especially when the two components had a fractional charge. Other cases where $\ell_1 = -\ell_2$ did not follow a clear rule.

In the case of half-integral vortex beams we saw the chains of alternating vortices move only slightly, deferring the larger portion of the motion to peripheral vortices as the ratio of intensities was varied. We observed that the chain of vortices of alternating sign disappeared only due to the local presence of a peripheral vortex.

Given the wealth of departures from the case of composite beams with integral components it is clear that an analytical theory is needed for a better understanding of this problem.

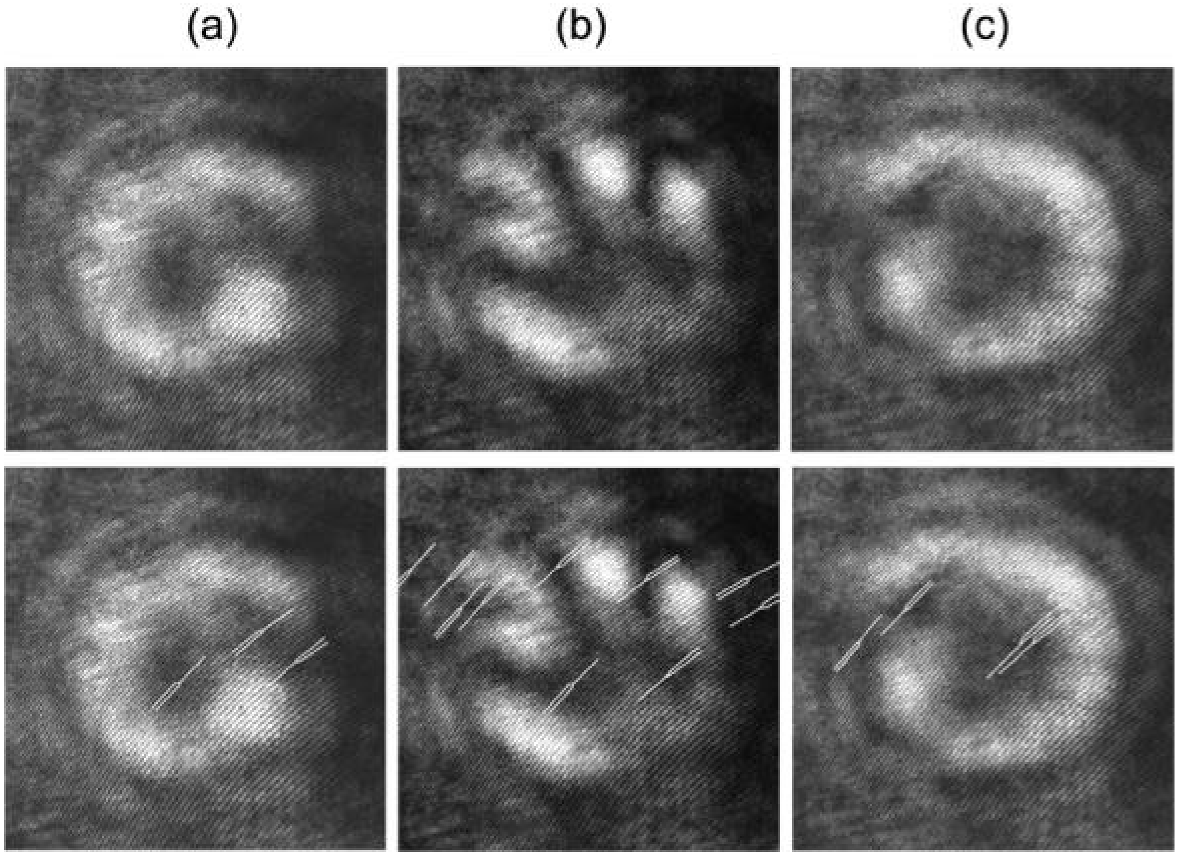

Figure 5. Interferograms for the case when the component beams had charges $\ell_1 = -1.5$ and $\ell_2 = +2.5$. The center column (b) corresponds to the case of equal intensity while the two ends correspond to each individual component, (a) for ℓ_1 and (c) for ℓ_2. The top row has the raw patterns. In the bottom row we label the forks that we identified, with tines pointing to the upper left (lower right) denoting positive (negative) topological charge.

ACKNOWLEDGMENTS

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REFERENCES

1. Galvez, E.J., Smiley, N., and Fernandes, N., "Composite optical vortices formed by collinear Laguerre-Gauss beams," *Proc. SPIE* **6131**, 19–26 (2006).
2. Maleev, I.D. and Swartzlander, G.A., Jr., "Composite optical vortices," *J. Opt. Soc. Am. B* **20**, 1169–1176 (2003).
3. Berry, M.V., "Optical vortices evolving from helicoidal integer and fractional phase steps," *J. Opt. A* **6**, 259–268 (2004).
4. Leach, J., Yao, E., and Padgett, M.J., "Observation of the vortex structure of a non-integer vortex beam," *New J. Phys.* **6**, 1–8 (2004).
5. Basistiy, I.V., Pas'ko, V.A., Slyusar, V.V., and Soskin, M.S., "Synthesis and analysis of optical vortices with fractional topological charges," *J. Opt. A* **6**, S166–S169 (2004).
6. Baumann, S. and Galvez, E.J., "Non-integral vortex structures in diffracted light beams," *Proc. SPIE* **6483**, 64830T (2007).