Proposal to produce two and four qubits with spatial modes of two photons

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ABSTRACT

This article presents a proposal for preparing photon pairs in states that are entangled in their spatial modes. The method allows the encoding of any pair of spatial modes that is desired, without restrictions; a procedure that up to now has been elusive. This method consists of three steps: preparing photon pairs in polarization-entangled states, filtering the spatial mode, and use of polarization interferometers with diffractive mode-encoding elements to effect entanglement swapping between polarization and spatial modes. An extension of the method consists of entangling polarization and spatial mode, allowing the preparation of a 4-qubit cluster state of two photons.

Keywords: Qubits, Entangled Photons, Spatial Modes

1. INTRODUCTION

Entangled states of light are unquestionable features of quantum mechanics that distinguish it from classical mechanics. They have been used to demonstrate quantum mechanical predictions about nature such as contextuality and nonlocality, and are a fundamental component of quantum information. Light, via the process of spontaneous parametric down-conversion (SPDC), is a medium that allows easy and efficient production of quantum states entangled in energy and momentum,[1] polarization,[2,3] and spatial modes.[4] The latter involve high-order spatial modes that arise from the conservation of orbital angular momentum (OAM) in SPDC,[5–7] where light is in a superposition of many states of opposite OAM:[8,9]

$$|\psi_u\rangle = \sum_{\ell} c_\ell |u_\ell\rangle_1 |u_{\ell_p-\ell}\rangle_2, \tag{1}$$

where $|u_\ell\rangle$ represents the spatial mode with topological charge ℓ, which has OAM $\ell\hbar$, ℓ_p is the topological charge of the pump beam, and c_ℓ is a complex coefficient. The entanglement is retrieved by projection of the state onto a subset of modes via forked gratings,[4] spiral phase plates,[9] sector plates,[10] or spatial light modulators.[11,12] Hyperentanglement of photons is highly desired because of the difficulty of producing entanglement of many photons. The use of spatial modes for entanglement is attractive because of the unlimited number of states that are available, opening a Hilbert space greater than two for the purpose of encoding quantum information.

The recent use of diffractive optics to mix polarization and spatial mode has enabled the possibility of entanglement swapping between polarization and spatial-orbital modes of topological charge $\ell = 2$.[13] A recent proposal involves engineering the spatial mode of the pump beam in SPDC to bias the coefficients c_ℓ of Eq. 1 in favor of desired modes.[14] However, the technology for implementing it is still not available.

Recently, I proposed a method to entangle any two spatial modes using interferometry.[15] The method involves producing photon pairs of same polarization via Type-I SPDC, sending them through unbalanced Franson-type interferometers (i.e., with one arm longer than the other one by more than the coherence length of the light), and using of timing discrimination to eliminate the signal from the incoherent conthributions of the unequal paths. Spatial modes are encoded in the arms of the interferometers via diffractive optical elements. The discriminated paths leave the state of the light in a non-separable superposition of spatial modes.[16] One could also view this method as time-energy to spatial-mode entaglement swapping. A drawback of the method is that it requires that the pump beam have a large bandwith, restricting it to continuous-wave (cw) laser pumps.

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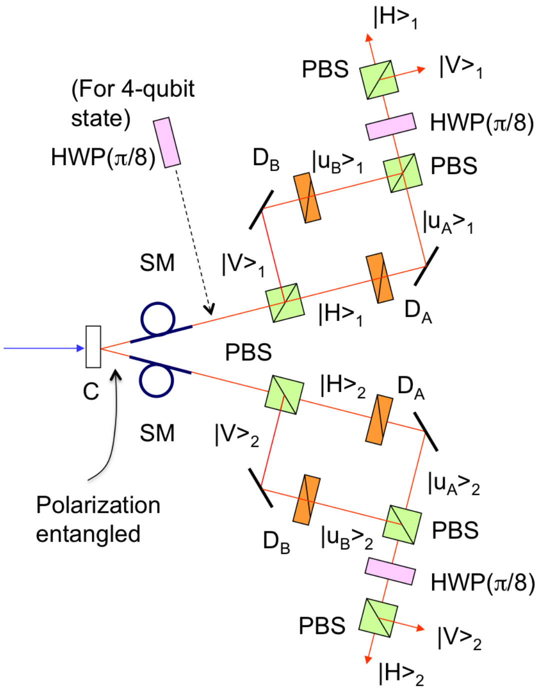

Figure 1. Schematic of the apparatus for the proposal. Optical elements shown are: polarizing beam splitters (PBS), non-linear SPDC crystal (C), single-mode fibers (SM), diffractive optical elements ($\mathrm{D_A}$, $\mathrm{D_B}$), half-wave plate with fast axis forming $\pi/8$ with the horizontal. Within kets are the polarization ($|H\rangle$ $|V\rangle$) and spatial-mode ($|u_A\rangle$ and $|u_B\rangle$) states of the light.

In this article I present a new method to entangle photon pairs in any desired spatial modes using polarization to spatial-mode entanglement swapping. It requires the production of polarization entangled states, but is not restricted to cw pump sources. The details of this method are described in Sec. 2. In addition, I present an alternative arrangement that involves entangling the polarization and spatial modes to obtain two qubits per photon. This proposal is presented in Sec. 3.

2. SPATIAL MODE ENTANGLEMENT

The starting point of this scheme involves producing polarization-entangled photon pairs, a method that is now quite mature.[2, 3] The polarization-entangled state produced by the SPDC source could be:

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|H\rangle_2 + e^{i\delta_0}|V\rangle_1|V\rangle_2\right)\otimes|\psi_u\rangle, \tag{2}$$

where δ_0 is a phase. Because the photons are in a superposition of spatial modes, we need to project these onto a single (fundamental) mode. We can do this by passing the photon pairs through single-mode fibers, as shown in Fig. 1.

Past the single-mode fibers the pairs are in the state

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_1|u_0\rangle_1|H\rangle_2|u_0\rangle_2 + e^{i\delta_0}|V\rangle_1|u_0\rangle_1|V\rangle_2|u_0\rangle_2\right), \tag{3}$$

where $|u_0\rangle$ is the (fundamental) spatial mode of the photons. The pairs can also be in the other polarization Bell states, depending on the source and optical components used.

Past the fibers, the optical components in each path effect entanglement swapping between polarization and spatial modes. This is done by passing each photon through a Mach-Zehnder interferometer that has polarization beam splitters. As is well known, if the polarization of the light in each arm is not changed, the light from both arms exits through the same port. However, distinct polarizations take different paths. We place diffractive elements in each arm so that a particular mode is associated with a state of linear polarization. For example, if the arm through which the horizontally-polarized photon goes has element D_A encoding spatial mode $|u_A\rangle$, and the arm through which the vertically-polarized photon goes has element D_B that encodes mode $|u_B\rangle$, then the light coming out of the interferometer is in the state

$$|\varphi'\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle_1 |u_A\rangle_1 |H\rangle_2 |u_A\rangle_2 + e^{i(\delta_0+\delta')} |V\rangle_1 |u_B\rangle_1 |V\rangle_2 |u_B\rangle_2 \right), \tag{4}$$

where δ' is the combined phase introduced by the two interferometers. After each interferometer we pass the light through a half-wave plate forming $\pi/8$ with the horizontal, performing the operations

$$\hat{\mathcal{H}}|H\rangle = |D\rangle \tag{5}$$

and

$$\hat{\mathcal{H}}|V\rangle = -|A\rangle, \tag{6}$$

where

$$|D\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle + |V\rangle \right) \tag{7}$$

and

$$|A\rangle = \frac{1}{\sqrt{2}} \left(-|H\rangle + |V\rangle \right). \tag{8}$$

The waveplates are followed by polarizing beam splitters. This way, the state after the two non-thru ports of the last polarizing beam splitters is

$$|\varphi_{\text{non-thru}}\rangle = \frac{1}{\sqrt{2}} |V\rangle_1 |V\rangle_2 \left(|u_A\rangle_1 |u_A\rangle_2 + e^{i(\delta_0+\delta')} |u_B\rangle_1 |u_B\rangle_2 \right) \tag{9}$$

and similarly, for the photons coming off the two thru ports the state is

$$|\varphi_{\text{thru}}\rangle = \frac{1}{\sqrt{2}} |H\rangle_1 |H\rangle_2 \left(|u_A\rangle_1 |u_A\rangle_2 + e^{i(\delta_0+\delta')} |u_B\rangle_1 |u_B\rangle_2 \right). \tag{10}$$

Both states given by Eqs. 9 and 10 are non-separable states of modes A and B. In effect, the interferometer creates spatial modes associated to the polarization states, and the components after the interferometers complete the swapping by removing the polarization labeling. An interesting result is that the phase δ_0 due to polarization entanglement (inserted by the source) and phase δ' due to mode entanglement (inserted by the interferometers) add. Thus we can tune the state more conveniently with the interferometers.

This scheme can be realized less expensively by using collinear down-conversion and only one interferometer. The apparatus would then need two additional non-polarizing beam-splitters to separate the pairs of photons. The modes can also be encoded by spatial light modulators (SLM). The novel aspect of this proposal is that we can have the freedom to encode any mode that we desire, and thus produce entangled states in high-dimensional mode spaces.

3. SPATIAL AND POLARIZATION ENTANGLED STATES: FOUR QUBIT CLUSTER STATES

Let us return to the state of the light before the interferometer, Eq. 2. We pass photon 1 through a half-wave plate forming $\pi/8$ with the horizontal. The state then becomes

$$|\varphi\rangle = \frac{1}{\sqrt{2}} \left(|D\rangle_1 |H\rangle_2 - |A\rangle_1 |V\rangle_2 \right), \tag{11}$$

where for simplicity we set $\delta_0 = 0$. Passage of the light through the interferometers leaves the light in the state

$$|\varphi\rangle = \frac{1}{2}\left(|H\rangle_1|u_A\rangle_1|H\rangle_2|u_A\rangle_2 + |V\rangle_1|u_B\rangle_1|H\rangle_2|u_A\rangle_2 + |H\rangle_1|u_A\rangle_1|V\rangle_2|u_B\rangle_2 - |V\rangle_1|u_B\rangle_1|V\rangle_2|u_B\rangle_2\right), \qquad (12)$$

where we also set $\delta' = 0$. The state of Eq. 12 is the four-qubit cluster state.[17, 18] Challenges associated with the use of this technique involve the need to project the state of modes. This can be accomplished with diffractive optics that convert the desired spatial mode onto the fundamental mode, and use a single-mode fiber as a filter that transmits only the fundamental mode.[4] SLMs constitute an important possibility for projecting and encoding spatial modes.[11, 12] A new possibility consists of using an OAM sorter, demonstrated recently.[19]

4. CONCLUSIONS

In summary, I present a new proposal for producing entangled states of any modes that are desired via polarization to spatial-mode entanglement swapping. The method is less restrictive than the previous proposal[15] and seems experimentally feasible. The implementation of methods of producing entangled states of spatial modes on demand has the potential to stimulate new schemes for hyperentangling light for quantum computation and for encoding quantum information.

The proposal of Sec. 2 would produce two-qubit states made exclusively of spatial modes. A simple variation of the scheme, described in Sec. 3, allows the creation of four-qubit states of two photons, and in particular a cluster state, which is a state that can be used as a starting point for implementing one-way quantum computation algorithms.[18] An experimental demonstration of these proposals is currently underway in our laboratory.

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