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Quantum partition noise in a superconducting tunnel junction

K. Segall* and D. E. Prober†
Departments of Applied Physics and Physics, Yale University, New Haven, Connecticut 06520-8284

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The theory of charge partition noise of quasiparticles in a superconducting tunnel junction is developed. The charge fluctuations are shown to have a significant contribution from partition noise that arises from the quantum superposition of the electron and hole character of the quasiparticles. These fluctuations are dominant at small bias voltage. The charge fluctuations are compared to the usual Poisson "shot noise" of the current. The implications for the design of energy-resolving single-photon detectors are explored.

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Partition noise is one of the most fundamental effects in quantum physics.[1] It arises when a quantum particle has two or more possible paths, or outcomes, from which to "choose" before encountering a detector that can tell which path or outcome has been chosen. It is the counterpart of interference effects in which the paths are "recombined" before the particle is detected. Partition noise of fermion particles manifests itself in a particularly clear manner in mesoscopic systems, where current flows through a quantum coherent region connecting two reservoirs.[1,2] Partition noise has recently been utilized to study the effective charge of the carriers in exotic conductors, including fractional quantum Hall systems[3] and Andreev structures in superconductors,[4,5] and to detect the fermion correlations.[1]

In this paper we develop an understanding of charge partition noise in superconducting tunnel junctions based on conventional (low T_c) superconductors. This partition noise arises from the mixed electron-hole character of the elementary excitations in the superconductor, the quasiparticles. We treat the case of quasiparticles produced by single-photon absorption, which gives a non-steady-state, nonequilibrium quasiparticle population. Each quasiparticle excitation is a quantum superposition of electron (negative charge, $-e$) and hole (positive charge, e).[5] When a quasiparticle tunnels from one superconductor to another (Fig. 1), it must choose to tunnel as an electron or as a hole, transferring negative or positive charge respectively.[6] This choosing results in charge fluctuations—the charge partition noise. The mixed character of the quasiparticles was seen in past experiments on the tunnel injection of charge, which produced a charged, steady-state, nonequilibrium population of quasiparticles.[7] However, such steady-state nonequilibrium effects do not access the charge partition noise.

The experiment we consider starts with the creation of N_0 quasiparticles in the left superconductor (say 10^4), by absorption of a photon of energy ε.[8,9] The temperature is $T=0$, so there are no thermally excited quasiparticles. The recombination time is much greater than the tunnel time, so all the quasiparticles tunnel across the barrier and then diffuse away into the superconductor on the right, which is semi-infinite in size.[10] One measures the magnitude of the charge, Q, collected by the superconductor on the right.[11] For an ensemble of such measurements, one determines the magnitude of the average charge, Q_{av}, and σ_Q, the rms deviation from this average. The junction is biased by a dc voltage, $V<(2\Delta/e)$. The energy 2Δ is the minimum energy to break a pair. The energy spectrum of the quasiparticle excitations on each side is shown in Fig. 1, in the excitation representation.[5]

The quasiparticles, in the left superconductor, are distributed in an energy range $\delta E \ll 2\Delta$. δE decreases with time due to phonon emission, so we take its value to be that at the mean tunnel time. The average occupancy of each quasiparticle state is $\ll 1$. The energy of a quasiparticle in a momentum state with wave vector magnitude k, relative to the Fermi energy E_F, is given by $E_k=(\xi_k^2+\Delta^2)^{1/2}$, with ξ_k the energy of the single-electron state in the normal metal relative to E_F. In a free-electron metal near k_F, $\xi_k=2E_F(k-k_F)/k_F$, and in general, ξ_k is antisymmetric about k_F, and proportional to $(k-k_F)$ near k_F. The fractional electron character of a quasiparticle is given by the quantity $u_k^2=(1/2)(1+\xi_k/E_k)$, and the fractional hole character is $v_k^2=(1-u_k^2)$. This "character" varies from holelike for k well below the Fermi wave vector k_F, to electronlike for $k \gg k_F$. At $k=k_F$, $u_k^2=v_k^2=(1/2)$; the character is equally electron and hole. For $V=0$, the probability of electron tunneling is u_k^2 from a given k state, and the probability of hole tunneling is $(1-u_k^2)$. For $V=0$, both electron and hole tunneling are allowed by energy considerations, for all states. Since there are two k states for each E_k, symmetrically below and above k_F, electron and hole tunneling are equally likely from these two E_k states.

The electron and hole tunneling processes at finite voltage are shown in Fig. 1. At finite voltage, energy restrictions affect how some or all of the quasiparticles can partition. Tunneling as an electron transfers a negative charge, $-e$, horizontally to the right, adding an electron to the total charge tunneled. E_k on the right is higher by eV than the starting energy E_k on the left, since the Fermi levels in this diagram are offset by the electrochemical potential difference, eV. The hole process, shown by dashed lines, transfers a negative electron charge from right to left, equivalent to a hole tunneling from left to right. For the hole process, E_k on the right is lower by eV than E_k on the left. (For this hole process the tunneling matrix element is the same as the electron process.)

The underlying electron-hole symmetry is apparent in Fig. 2, where we employ a modified semiconductor representation.[12] A quasiparticle in the upper band is a filled

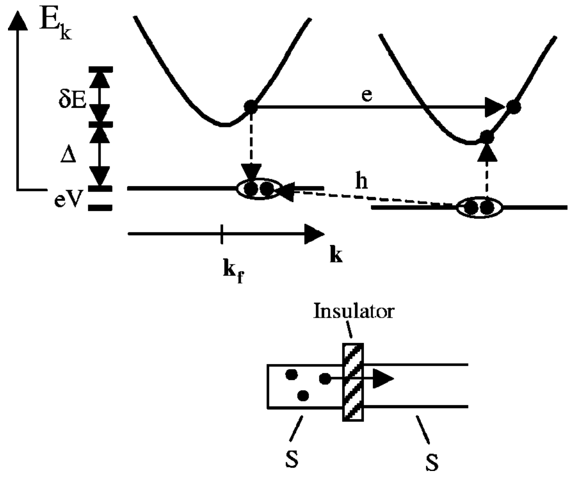

FIG. 1. Excitation representation of the superconducting tunnel junction. The vertical energy axis is for the left superconductor. Pairs are shown by double dots.

circle, in the lower band an open circle. Each quasiparticle has a mixed electron-hole character. The quasiparticles are labeled in this diagram only by E_k, and thus represent both k states, above and below k_F. The energy E_k in the upper band increases in the upward direction, and E_k in the lower band increases in the downward direction. For both bands the zero of energy is midway between. A photon absorption process creating two quasiparticles for a photon of energy $\varepsilon > 2\Delta$ is shown by the dashed vertical arrow. The energy change upon tunneling is shown by a vertical displacement of eV upon tunneling. Quasiparticles increase in energy by eV upon tunneling as electrons, and decrease by eV if tunneling as holes. These energy considerations affect the partitioning. Quasiparticles with $E_k > (\Delta + \mathrm{eV})$ can tunnel as either electrons or holes. The choosing (partitioning) between electron or hole tunneling is governed by a quantum probability, and this gives the variation of the collected charge from one measurement to the next. We also see that a quasiparticle cannot tunnel as a hole if $E_k < (\Delta + \mathrm{eV})$, because the final energy would be in the gap on the right, with no available states.

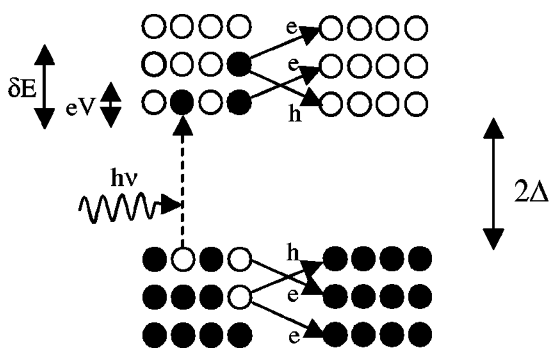

FIG. 2. Modified semiconductor representation of the tunneling process, in which electron and hole tunneling processes and their directions are shown. These include a vertical shift by eV or $-$eV, depending on the process. For the lower band, energy is measured in the downward direction. Hole tunneling from left to right is forbidden for $E_k < (\Delta + eV)$.

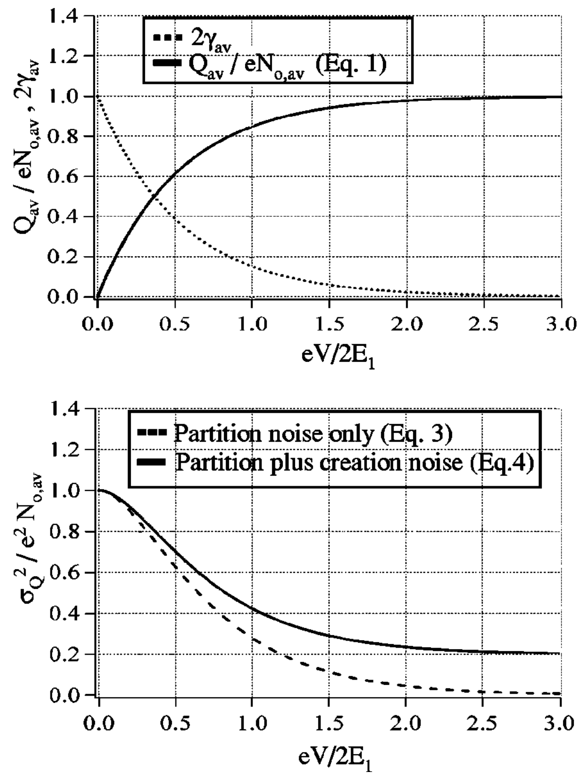

FIG. 3. Magnitude of the average charge, average hole tunneling probability (γ_{av}), and charge variance $(\sigma_Q)^2$ versus voltage.

Only electron tunneling is allowed for these quasiparticles; no partitioning is possible. Hole tunneling from left to right is fully suppressed when $\mathrm{eV} > \delta E$.

We can now consider the voltage dependence of the charge. N_0 quasiparticles are created in the left superconductor in one measurement. For an ensemble of measurements, the average number of quasiparticles created in one measurement is $N_{0,\mathrm{av}}$. The magnitude of the charge collected, in a specific measurement, is $Q = e(N_e - N_h) = eN_0(1 - 2\gamma)$, with N_e (N_h) the number of quasiparticles which tunnel as electrons (holes), and γ the fraction of quasiparticles that tunnel as holes in that measurement.

The magnitude of the average charge collected is

$$Q_{\mathrm{av}} = eN_{0,\mathrm{av}}(1 - 2\gamma_{\mathrm{av}}). \tag{1}$$

For $V = 0$, on average half the quasiparticles tunnel as electrons and half as holes, so $\gamma_{\mathrm{av}} = 1/2$ and $Q_{\mathrm{av}} = 0$. The probability to tunnel as a hole decreases as the bias voltage is increased from $V = 0$ (see Fig. 3). For $eV > \delta E$, $\gamma_{\mathrm{av}} = 0$ and $Q_{\mathrm{av}} = eN_{0,\mathrm{av}}$, as all quasiparticles must tunnel as electrons. The quantity γ_{av} depends on the bias voltage and can be calculated if the energy distribution is known. In Fig. 3 we plot γ_{av} and Q_{av}, with γ_{av} determined for a Fermi-Dirac distribution with an effective quasiparticle temperature $T_1 = E_1/k_B$. In Fig. 3 we see that Q_{av} is nearly $eN_{0,\mathrm{av}}$ for $eV > 6E_1$; thus, δE is about $6E_1$. (In our past experiments, the quasiparticle energy distribution was inferred to be approximately thermal, with T_1 about 0.6 K, higher than the lattice temperature.[9])

The partitioning between electron and hole tunneling, along with the randomness of the occupancy of the quasiparticle states, gives the charge fluctuations for an ensemble of measurements. This is equivalent to choosing γN_0 quasiparticles to be holes from the total of N_0 in that measurement, with a selection probability γ_{av}.[13] The fluctuations of the number of holes, γN_0 is given by the variance of the binomial distribution for γ with N_0 trials:

$$(\sigma_\gamma)^2 = \gamma_{av}(1-\gamma_{av})/N_0. \tag{2}$$

Variations in γ lead to variations in Q. Q would also vary if there were a time-varying imbalance in the occupancies of the electronlike ($k>k_f$) and holelike ($k<k_f$) branches.[5,7] However, significant branch imbalance does not develop, because the branch imbalance relaxation time is typically much shorter than the tunnel time. For example, in our Al tunnel junctions,[9] the tunnel time is $\sim 10^{-6}$ s, whereas the branch mixing time is of order 10^{-8} s.[7] The variance of Q for fixed N_0 is thus given simply by $(\sigma_Q)^2=(\sigma_\gamma)^2(dQ/d\gamma)^2 = 4e^2N_0^2(\sigma_\gamma)^2$; therefore,

$$(\sigma_Q)^2 = 4e^2N_0\gamma_{av}(1-\gamma_{av}). \tag{3}$$

This result is plotted in Fig. 3. The maximum charge noise, σ_Q, is at $V=0$ where $\gamma_{av}=1/2$, even though $Q_{av}=0$. The mixed electron-hole character of the quasiparticles causes the large charge noise at zero bias. If quasiparticles were pure electrons and pure holes, as is the case for a true semiconductor, and were created in equal number by photoexcitation, one would collect exactly zero charge at $V=0$, with *no* fluctuations. A semiconductor does not display charge partition noise.

We now consider how the fluctuations of N_0 for the ensemble of measurements add to σ_Q, since usual methods of creating quasiparticles have fluctuations of N_0. We refer to these as creation statistics. For photon excitation, the conversion of photon energy to quasiparticle number does not happen exactly the same way every time: some of the energy goes into phonons (lattice excitations) instead of going into quasiparticles (electronic excitations). This division has variations from one measurement to the next. We assume that fluctuations of N_0 and of γ are uncorrelated, i.e., the total charge variance is $(\sigma_Q)^2=(\sigma_{N_0})^2(dQ/dN_0)^2 + (\sigma_\gamma)^2(dQ/d\gamma)^2$. It is found that $(\sigma_{N_0})^2=FN_{0,av}$, where F is the Fano factor, typically$=0.2$.[8] We find that

$$(\sigma_Q)^2 = 4e^2N_0\gamma_{av}(1-\gamma_{av}) + Fe^2N_{0,av}(1-2\gamma_{av})^2. \tag{4}$$

Note that F can also include contributions from sources other than creation statistics, making it larger than the value of 0.2. Examples of such contributions are quasiparticle multiplication due to trapping[9] or recombination.[9,10] We treat here the case of only creation statistics, with $F=0.2$. The total charge fluctuations have a value of $0.2\,e^2N_{0,av}$ at high bias, for $eV>\delta E$; there is no contribution from partition noise. For smaller voltages, the partition noise does contribute, and the contribution to Eq. (4) due to the creation statistics decreases. At $V=0$ we have $\gamma_{av}=1/2$, and only the partition noise contributes to the charge variance.

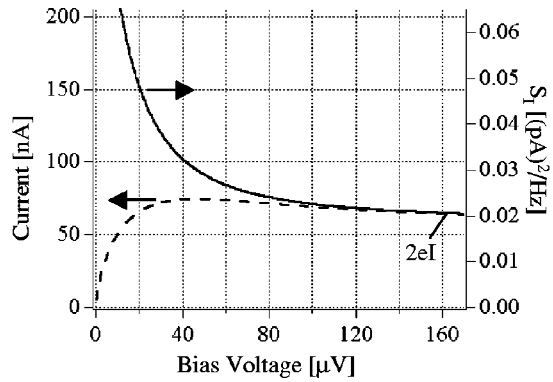

FIG. 4. Magnitude of the BCS thermal current and the associated low-frequency current noise vs voltage, for an aluminum tunnel junction at $T=0.25$. The current spectral density approaches the shot noise limit of $2eI$ at high voltage, and the Johnson-Nyquist limit of $4kTG$ at low voltage.

The charge partition noise is, in some respects, similar to the *current* noise in an *SIS* tunnel junction at finite temperature,[6] with quasiparticles in a thermal distribution on both sides of the barrier. At a finite voltage a steady-state current flows. We assume the barrier transmission is small so that multiparticle tunneling can be neglected. The previous arguments concerning the energy dependencies of tunneling by electrons and holes are still valid, with the energy spread δE being of order $6k_BT$. The predicted I-V curve[14] is shown in Fig. 4 for an aluminum tunnel junction of normal resistance 1 Ω, at $T=0.25$ K. At $V=0$ there is an equal current of electrons and holes tunneling from each side, so the net current is zero. As V is increased the hole current from left to right and the electron current from right to left are reduced; hence a net electron current flows from left to right, and increases with voltage. At high voltages ($eV \gg k_BT$) hole tunneling from left to right and electron tunneling from right to left are prohibited. The average current approaches a nearly constant value. In the region in between, near $eV=2k_BT$, the current has a weak maximum due to the structure of the BCS density of states.[5,14]

The spectral density of the low-frequency ($f \ll eV/h$) current fluctuations, in A^2/Hz, is given by[6]

$$S_1 = 2eI\coth(\mathrm{eV}/2k_BT), \tag{5}$$

which also plotted in Fig. 4. Equation (5) also applies for a nonsuperconducting tunnel junction, so a measurement of S_I does not access the partition noise of quasiparticles. At large voltage ($eV \gg k_BT$), only one type of charge tunneling is allowed from each side, and one recovers the usual Poisson shot noise result:[1,6]

$$S_1(V \gg k_BT/e) = 2eI. \tag{6}$$

I is the dc current. This noise, Eq. (6), arises from the random times at which the charges cross the barrier. At low voltage the noise increases above the Poisson shot noise value, due to the fact that both electron and hole currents are flowing from each electrode. At $V=0$, the predicted singularity of (I/V) is rounded out by finite lifetime effects,[6] giving a linear conductance G. One then obtains Johnson-Nyquist noise, in accordance with the fluctuation-dissipation theorem:[15] $S_I(V=0)=4k_BTG$, where G is the differential conductance and $dI/dV=I/V$. This shows that the Johnson-Nyquist noise and the shot noise arise from the same physics in these systems. At low voltage, S_I is due to both electron and hole currents from each side.

The charge noise (Fig. 3) and the current noise (Fig. 4) are similar in that both are maximum at $V=0$ and both are suppressed at high voltage. At high voltage, however, the partition noise [Eq. (3)] is reduced to zero while the current noise approaches a constant, nonzero value. This is because the current noise at high voltage is due to the randomness of traversal time for the charge to cross the barrier. For the charge collection experiment, in contrast, randomness of traversal time is not relevant, since one waits until all the quasiparticles have tunneled before the charge is recorded.

The predictions for charge partition noise can be studied experimentally in single-photon detectors based on the tunnel junction.[8,9,16] Here, a photon is absorbed in the left electrode, and $N_{0,\mathrm{av}}$ is proportional to the photon energy ε. Typically, $N_{0,\mathrm{av}}=0.6(\varepsilon/\Delta)$. The charge fluctuations, Eq. (4), limit the accuracy with which the photon energy can be determined. To reduce σ_Q the detector should be biased at a voltage such that $eV>\delta E$. To improve upon the Al devices previously studied,[8,9] one will need to reduce δE, by having the quasiparticles ‘‘cool’’ longer prior to tunneling, or by having faster phonon emission for better cooling. We have used a longer tunnel time, the first approach. One might also employ a tunnel junction material with stronger electron-phonon coupling.

The predictions we develop may also be useful in a range of other physics studies, for mesoscopic superconductor structures including Andreev systems,[1,5] and tunneling structures between a superconductor and another system with a gap (e.g., a magnet). In other systems the nature of the electronic excitations may be unknown, and studies of partition noise may elucidate their character. Many such systems, such as carbon nanotubes, conducting polymers, and DNA molecules, are now studied by tunneling experiments.[17]

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*Present address: MIT, 13-3010, Cambridge, MA 02139. Email address: ksegall@mit.edu

†Email address: daniel.prober@yale.edu

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[11] We do not consider the Josephson pair current, as it is fully suppressed by a magnetic field.

[12] The diagram of Fig. 2 differs from the simple semiconductor representation in which excitations in the upper band are electrons and in the lower band are holes. That simplified representation is adequate only for treating the current and current noise of the superconducting tunnel junction, with thermal equilibrium in each superconductor (Ref. 6). It is not adequate for treating the charge and its fluctuations.

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