A Theory of Racial Diversity, Segregation, and Productivity[*]

Chad Sparber

Colgate University

13 Oak Drive

Hamilton, NY 13346

csparber@mail.colgate.edu

Abstract

Empirical evidence illustrates that diversity generates both economic costs and benefits. This paper develops a theoretical model that accounts for the positive and deleterious effects of heterogeneity. First, an expanded Solow Growth Model demonstrates that the direct effects of diversity can be positive or negative, and depend upon the size of fixed parameter values. Second, diversity also influences individuals' location decisions. Segregation (variation of diversity across regions) always reduces national output per worker, so if diversity induces integration, it indirectly augments productivity as well. Finally, political policies aimed at reducing interaction costs across groups may actually reduce aggregate output per worker by encouraging segregation.

Key Words: Diversity, Segregation, Macroeconomic Productivity, Growth and Development Theory

JEL Classification Codes: O40, J24, O51, J10

1   Introduction

A growing body of empirical growth literature is assessing whether racial (or ethnic) diversity leads to macroeconomic gains or losses. The earliest economic analyses typically found evidence for the former. Groups often organize according to ethnic or racial identities to compete for rents, public goods, and favored political policies that detract from productive activity. In the extreme, diversity can lead to social conflict, exploitation, and violence. Quantitatively, Easterly and Levine (1997) argue that "a movement from complete heterogeneity to complete homogeneity... is associated with an income increase of 3.8 times."[1]

Recent work, in contrast, highlights the gains from diversity. Ottaviano and Peri (2006) find that cultural diversity (based upon immigrants' countries of origin) complements production and boosts native-born wages and productivity in US cities. Sparber (2007) analyzes US industries and argues that racial diversity generates productivity gains for most sectors of the economy. The effects are particularly large for industries employing highly educated workers, which suggests that diversity complements the decision makers of the labor force.[2]

This paper develops a cohesive theory to understand the net macroeconomic effects of diversity. I begin with a standard Solow production function and expand it by introducing two key features to demonstrate that the direct effects of diversity may be positive or negative. First, the model recognizes the possible existence of complementarities between workers of different types. Second, it includes an expression to capture the costs of communication, interaction, and conflict across groups. A simple relationship between the costs of cross-group communication and potential complementarities determines whether the direct effects of diversity result in macroeconomic gains or losses.

Next, I introduce a utility function that assumes individuals prefer to live among members of their own group. Segregation arises when regions have differing levels of diversity, yet agents have no incentive to engage in interregional migration. That is, people may be indifferent between living in a region that offers high wages and another region that maintains a larger population of individuals from their own group. If minorities reside in each region, then maximum aggregate output per worker occurs when groups fully integrate across regions. Segregation therefore reduces productivity.

Diversity plays a critical role in determining the location and migration decisions of individuals and thus indirectly affects productivity as well. Segregation tends to occur when minorities comprise a small share of the labor force. Complementarities imply that an

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[1] Alesina et al (2003) provide a recent replication of these results. Also see Alesina and La Ferrara (2005), Fukuyama (1999), Knack and Keefer (1997), and Mauro (1995).

[2] For evidence in the management literature, see O'Reilly, Williams, and Barsade (1998), Cox and Blake (1991), or Cox, Lobel, and McLeod (1991).

individual could earn a higher wage by moving to the region in which members from his/her own group are more scarce, but the costs of interaction are sufficiently low when little diversity exists that own-group preferences outweigh potential wage gains. Costs become more apparent as diversity increases. This encourages individuals to seek higher wages and integrate.

Exogenous policy-driven costs of diversity operate in a similar way. Segregation arises when the cost of interaction with other groups is low since minorities are willing to sacrifice wage gains in order to live and work among other minorities. When interaction becomes more difficult, the wage penalty from forming such enclaves rises and groups integrate. Since segregation always reduces productivity, political policies encumbering communication between groups can ironically promote productivity by encouraging integration.

The net macroeconomic effects of diversity critically depend upon whether minority groups evenly distribute themselves across regions or instead form segregated clusters. The ramifications of the theoretical model mandate that cross-country empirical assessments of diversity and productivity also consider the role of segregation. In the absence of reliable segregation data, however, alternative methods must be employed to understand the magnitude of diversity's productivity consequences. Therefore, I summarize my analysis by simulating an economy with parameter values that characterize the United States. Results imply that the increase in racial diversity among US college-educated workers between 1990 and 2000 may have augmented output per worker by 0.6% and that this diversity led to a level of productivity that is also 0.6% higher than what would occur in a homogenous society.

2   Production and the Direct Effects of Diversity

2.1   Production Function

Many economists have developed theories of diversity and production, but few have sought to capture both the costs and benefits of heterogeneity. Caselli and Coleman (2002) create a model of greed-motivated conflict to explain the prevalence of ethnic tension. Individuals organize into groups that compete over a country's wealth. Conflict escalates when losers find it difficult to switch alliances and capture the spoils of war, as is the case when groups organize according to racial or ethnic identities. The authors do not, however, recognize any benefits of employing a heterogeneous workforce.

Conversely, Page (2007) and Hong and Page (2004) focus on the problem solving capabilities of a workforce and argue, "A collection of problem solvers of diverse abilities tends to jointly outperform a collection of high quality problem solvers... Problem solvers differ along

two dimensions: their encodings of problems, *perspectives*, and the algorithms they apply in searching for solutions, *heuristics*." They assume that the best problem solvers tend to take similar approaches when encountering tasks, but that diversity aids the decision making process by bringing new perspectives and heuristics to a task.

Ottaviano and Peri (2005) provide a valuable model that recognizes both the positive and negative effects of diversity.[3] First, groups complement each other in production according to a Dixit-Stiglitz love-of-variety term $\left(\sum_{i=1}^{N} L_i{}^{\alpha}\right)$. For every unit of labor supplied, however, only a fraction $(1-\tau)$ is available for production. They interpret this as a resource cost of interaction (or communication) between groups. Interaction costs, production complementarities, exogenous capital (K), and technology (A) combine to produce output (Y) according to Equation (1).

$$Y = F(K, L_1, ..., L_N) = AK^{1-\alpha}(1-\gamma)^{\alpha} \sum_{i=1}^{N} L_i{}^{\alpha} \tag{1}$$

I advocate a model of production that modifies (1) but preserves three features. First, it must allow for the possibility of complementarities between workers of different types. Second, it should include a term to capture the costs of communication, interaction, and conflict across groups. Third, an ideal model of diversity and productivity should nest more traditional production theories within it.

Suppose a country is divided into N unique labor groups.[4] All agents work so that the population and employment of group i both equal L_i. Members from varied groups complement each other in production according to $C(L_1, ..., L_N, \sigma, \alpha)$, where σ is an exogenous parameter that determines the degree of substitutability between groups and α is the share of output paid to labor. Though potential complementarities exist, interaction and communication across groups may be costly. Firms retain and sell only a fraction of output, $R(L_1, ..., L_N, \gamma)$, where γ is an exogenous cost parameter.[5] These factors combine with exogenous capital (K) and technology (A) to produce output (Y) according to the general function in (2).

$$Y = AK^{1-\alpha} \cdot R(L_1, ..., L_N, \gamma) \cdot C(L_1, ..., L_N, \sigma, \alpha) \tag{2}$$

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[3] Also see Alesina, Spolaore, and Wacziarg (2005), Alesina and La Ferrara (2005), and Lazear (1999).

[4] Groups may be differentiated by any factor, including race, ethnicity, gender, age, educational attainment, etc. This paper, however, will focus on the role of race.

[5] Many empirical papers note that institutional factors such as the system of government or stage of development can play an important role in determining the net effects of diversity. (See Collier (2000), Collier (2001), and Alesina and La Ferrara (2005)). Thus, exogenous parameter values may vary across countries.

Consider the complementarity and cost components of (2) in turn. Economists have long adopted production functions in which an increase in the variety of intermediate capital inputs will augment output.[6] Equation (2) instead proposes that a variety of labor inputs will enhance productivity. Ottaviano and Peri (2005) adopt a similar philosophy and model labor complementarities with a love-of-variety term, $C(\cdot) = \sum_{i=1}^{N} L_i{}^\alpha$. More variety (greater N) always increases output for a given labor force size, and output is maximized if each group contributes the same number of workers. As an alternative, I employ the CES complementarity term in Equation (3) for fixed N. The advantage of this specification is that labor types may be grossly substitutable or complementary.[7] As σ approaches negative infinity, workers from different groups become perfect complements. When $\sigma = 0$, labor types enter the production function as Cobb-Douglas. Finally, $\sigma = 1$ represents the standard production assumption that labor types are perfect substitutes.

$$C(L_1, ..., L_N, \sigma, \alpha) = \left(\sum_{i=1}^{N} L_i^\sigma \right)^{\frac{\alpha}{\sigma}}, \tag{3}$$
$$\text{where } \sigma \in (-\infty, 1]$$

Assume a "semi-symmetric" labor force demography with one majority and $N-1$ minority groups. Each minority group comprises a share of the labor force equal to Θ, leaving the majority with a share M equal to $1 - (N-1)\Theta$. (Also assume that $\Theta < \frac{1}{2 \cdot (N-1)}$ to ensure a majority group exists). An increase in Θ implies that the minority share of the labor force comes closer to that of the majority and that diversity rises. Under these semi-symmetric assumptions, the complementarity expression in (3) reduces to Equation (4). The marginal effects of increasing Θ are positive so long as $\Theta < \frac{1}{N}$.[8] Increasing the labor force share of minority groups will cause labor complementarities to rise for $\sigma < 1$.

$$C(\cdot) = L^\alpha \cdot ((N-1) \cdot \Theta^\sigma + M^\sigma)^{\frac{\alpha}{\sigma}} \tag{4}$$

$$\frac{\partial C(\cdot)}{\partial \Theta} = \alpha \cdot (N-1) \cdot L^\alpha \cdot (\Theta^{\sigma-1} - M^{\sigma-1}) \tag{5}$$
$$\cdot ((N-1) \cdot \Theta^\sigma + M^\sigma)^{\frac{\alpha-\sigma}{\sigma}}$$

The production function in (2) also accounts for the costs of diversity. Again, I follow

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[6] See Ethier (1982) and Romer (1987) for early examples.

[7] The degree of substitutability is likely to vary across industries and countries (see Sparber (2007)). However, this paper will hold σ constant to facilitate better understanding of the effects of diversity.

[8] The assumption that a majority group exists assures that this condition holds.

the lead of Ottaviano and Peri (2005) who measure retained output by including a multiplicative term ranging from zero to one, $R(\cdot) = (1 - \gamma)$. The parameter γ is open to broad interpretation – it could reflect the cost of offering employees language classes or formal diversity training, prevailing tension and conflict across groups, or the expense of offering group-specific public goods.[9] Also note that it is not a function of the total size of the labor force, so there are no scale effects to diversity costs. Unfortunately, however, by expressing costs as a multiplicative constant, the quantity is unresponsive to a country's demographic shifts. That is, firms bear communication costs even if all workers belong to the same group.

Instead, I adopt the cost expression in (6). Note that costs reduce to $\exp\left(-\gamma \cdot \prod_{i=1}^{N} \theta_i\right)$, where θ_i is the labor force share of group i, so that diversity costs remain impervious to scale effects. More importantly, the function assumes that exogenous interaction costs disproportionately encumber diverse societies. That is, if language classes become more expensive, it will mostly affect firms training a highly diverse workstaff. If inter-group tensions rise, heterogenous countries would experience vast losses. The quantity of output retained by the firm is maximized when one group comprises the entire labor force. It is minimized when all groups have an equal labor force share.[10]

$$R(L_1, ..., L_N, \gamma) = \exp\left(\frac{-\gamma \cdot \prod_{i=1}^{N} L_i}{\left(\sum_{i=1}^{N} L_i\right)^N}\right), \tag{6}$$

$$\text{where } \gamma \geq 0$$

Equation (6) guarantees that the costs of interaction increase in γ. Small γ values approximate the typical production assumption of $\gamma = 0$, and large values are necessary to generate significant losses of output. Political policies are likely to affect γ, which may vary

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[9]See Lazear (1999) for a model of diversity and language. See Easterly and Levine (1997) and Alesina and La Ferrara (2005) for empirical and theoretical evidence on diversity and public goods provision, respectively.

[10]An alternative cost expression with qualitatively similar results would specify $R(\cdot) = \exp\left(-\gamma \cdot \left(1 - \frac{\sum_{i=1}^{N} L_i^2}{\left(\sum_{i=1}^{N} L_i\right)^2}\right)\right)$, which reduces to $R(\cdot) = \exp\left(-\gamma \cdot \left(1 - \sum_{i=1}^{N} \theta_i^2\right)\right)$. The advantage of this approach is that γ is immediately interpretable as the coefficient of the "ethnolinguistic fractionalization index" typically estimated in the empirical literature (assuming that $\sigma = 1$). Unfortunately, however, many of the conclusions in Section 3.2 become less clear. Further information about this methodology is available upon request.

Both the ethnolinguistic fractionalization index and the expression in (6) implicitly assume that cultural distance and language differences between any two groups will not depend upon the groups considered. For empirical counter-examples, see Ginsburgh and Weber (2005), Hofstede and Hofstede (2005), or Inglehart, Basanez, and Moreno (1998).

across countries. Later sections of the paper will assess the ramifications of such actions.

The log-linear nature of (6) has particularly important implications. First, it facilitates closed form solutions that permit key insights throughout the paper. Second, values are easy to interpret: $\gamma \cdot \prod_{i=1}^{N} \theta_i$ represents the percentage of output lost due to diversity, so γ is the marginal percentage of output lost from a change in demography. For example, assume the semi-symmetric case holds so that retained output reduces to Equation (7). Equation (8) indicates that increasing the size of each minority group delivers detrimental consequences for the portion of output retained by the firm as the costs of interaction, communication, and conflict rise. Further suppose that $N = 5$ groups exist and that each of four minority groups maintain 5% of the labor force (leaving the majority group with the remaining 80%). Equation (7) implies that firms lose approximately 0.5% of output for every 1000-unit increase in γ (see Figure 1).

$$R(\cdot) = \exp\left(-\gamma \cdot \Theta^{N-1} \cdot M\right) \tag{7}$$

$$\frac{\partial R(\cdot)}{\partial \Theta} = -\gamma \cdot (N-1) \cdot (1 - N\Theta) \cdot \Theta^{N-2} \tag{8}$$
$$\cdot \exp\left(-\gamma \cdot \Theta^{N-1} \cdot M\right)$$

Appropriate values of γ are debatable. If special training programs are the only cost of diversity, and each employee invests 1.25 workdays of a 250-day workyear to training, then $\gamma = 1000$. Alternatively, Easterly and Levine (1997) estimate that a movement from complete homogeneity to a level of diversity with $N = 5$ and $\Theta = 0.05$ will be associated with a 47% decline in output. This implies $\gamma = 94,000$ (if $\sigma = 1$), though it is doubtful costs are this high in developed countries. In any case, it is clear that γ can represent high values and a country will still be able to retain significant levels of its output.

The complementarity term in (3) and cost expression (6) combine to form the production function in Equation (9), which satisfies the criteria established at the beginning of this section. It exhibits constant returns to capital and labor, allows for both interaction costs and potential complementarities across types of workers, and reduces to the standard Solow production function with exogenous technological growth in the special case of $\gamma = 0$ and $\sigma = 1$.

$$Y = F(K, L_1, ..., L_N) = A \cdot K^{1-\alpha} \cdot \exp\left(\frac{-\gamma \cdot \prod_{i=1}^{N} L_i}{\left(\sum_{i=1}^{N} L_i\right)^N}\right) \cdot \left(\sum_{i=1}^{N} L_i^{\sigma}\right)^{\frac{\alpha}{\sigma}} \tag{9}$$
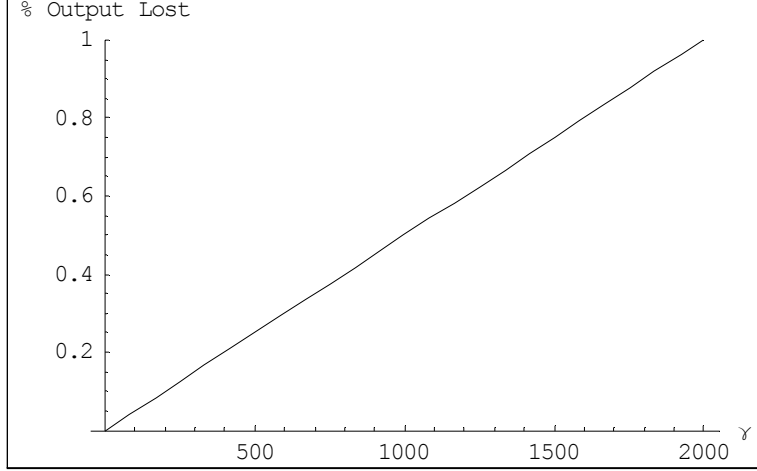
Figure 1: Interaction Costs and Lost Output. Output declines by 0.5% for every 1000 unit increase in γ, assuming $N = 5$ groups exist and each minority group maintains 5% of the labor force.

Equation (10) describes output per worker under the semi-symmetric assumptions. Increasing the size of minority groups has two opposing effects. On the one hand, a large minority presence requires firms to expend effort and energy facilitating inter-group communication, which causes productivity $(y \equiv \frac{Y}{L})$ to fall. On the other hand, diversity will generate production gains due to complementarities between workers.

$$
\begin{aligned}
y \;=\; & A\left(\frac{K}{L}\right)^{1-\alpha} \\
& \cdot \exp\left(-\gamma\Theta^{N-1}M\right) \\
& \cdot \left((N-1)\Theta^{\sigma} + M^{\sigma}\right)^{\frac{\alpha}{\sigma}}
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\frac{\partial y}{\partial \Theta} \;=\; & \left(\frac{y\cdot(N-1)}{(N-1)\Theta^{\sigma}+M)^{\sigma}}\right) \\
& \cdot\left(\alpha\cdot\left(\Theta^{\sigma-1}-M^{\sigma-1}\right)\right. \\
& \left.-\gamma\cdot(1-N\Theta)\left((N-1)\Theta^{\sigma}+M^{\sigma}\right)\Theta^{N-2}\right)
\end{aligned}
\tag{11}
$$

Not surprisingly, the relationship between γ (the conflict parameter) and σ (the complementarity parameter) characterizes the direction of diversity's direct effect on productivity. If groups are largely substitutable for one and other and costs are high, diversity is likely to be detrimental. Conversely, benefits will supersede costs if minority participation complements the abilities of the majority workforce in problem-solving and decision-making

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processes. Given the assumptions made thus far, the direct effect of diversity will be positive if Inequality (12) holds.[11]

$$\gamma \;<\; \left(\frac{\alpha}{(1 - N \cdot \Theta)\, \Theta^{N-1} M} \right) \\ \cdot \left(\frac{M\Theta^\sigma - \Theta M^\sigma}{(N-1) \cdot \Theta^\sigma + M^\sigma} \right) \tag{12}$$

3 Utility and Labor Mobility

3.1 Utility

The production function in (9) will produce many interesting results and key insights into the direct macroeconomic effects of diversity, but if diversity influences individual location and migration decisions, it will indirectly affect aggregate output as well. The analysis now turns to a more formal exploration of labor mobility, an explicit indirect utility specification, and a more nuanced series of productivity ramifications.

Suppose a country has two geographically large regions so that agents must produce output in the region of their residence.[12] Assume that each region maintains an equal technology level and capital-labor ratio. Individuals earn utility from two sources. First, agents seek increased real wages. I measure the real wage as the marginal product of labor, adjusted by a cost of living equal to the marginal product of capital.[13] Second, Marsden's (1988) sociological account of "Homogeneity in Confiding Relations" notes that individuals prefer to communicate, socialize, and live with members of their own race.[14] Thus, an increase in the population (or labor force) share of a person's racial group causes his or her utility to rise. Altogether, $U_{j,k} = U\left(\theta_{j,k}, \frac{MPL_{j,k}}{MPK_{j,k}}\right)$ represents a generic indirect utility

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[11] Assuming that agents earn a wage equal to their marginal productivity of labor, this model implies that minorities will earn more than majority workers if and only if Inequality (12) holds. Thus, the model can explain why minorities often earn low wages without appealing to discrimination or intrinsic productivity differences across groups. I caution that, in reality, discrimination probably exists. If so, it may be possible for minorities to earn lower wages regardless of whether diversity generates macroeconomic productivity gains. Thus, I view the rigid link between the marginal effects of diversity and relative wages as a limitation, rather than a strength, of the model.

[12] Since individuals could live in one city or neighborhood and work in another, this model is inappropriate for examination of neighborhood or city segregation. Instead, see Cutler and Glaeser (1997) or Glaeser, Scheinkman, and Shleifer (1995).

[13] To ensure all individuals earn positive wages, assume $\gamma < \dfrac{\alpha \cdot \theta_j^\sigma}{(1 - N \cdot \theta_j)\left(\prod\limits_{i=1}^{N} \theta_i\right)\left(\sum\limits_{i=1}^{N} \theta_i^\sigma\right)}$ for all $j = 1, ..., N$ such that $\theta_j < \frac{1}{N}$.

[14] Also see Akerlof and Kranton's (2000) work on "Economics and Identity," or Briggs's (2002) account of race, segregation, and social capital.

function (increasing in each argument) for a person of Type j working in Region k. I adopt the specific utility function in Equation (13).

$$
\begin{aligned}
U_{j,k} &= \theta_{j,k} \cdot \frac{MPL_{j,k}}{MPK_{j,k}} \\
&= \left(\frac{K_k}{L_k}\right) \cdot \left(\frac{\alpha \cdot \theta_{j,k}^{\sigma} - \gamma \cdot (1 - N \cdot \theta_{j,k}) \left(\prod_{i=1}^{N} \theta_{i,k}\right) \left(\sum_{i=1}^{N} \theta_{i,k}^{\sigma}\right)}{(1 - \alpha) \cdot \sum_{i=1}^{N} \theta_i^{\sigma}}\right)
\end{aligned}
\tag{13}
$$

Equilibrium requires individuals of Type j to have equal utility across regions so that $U_{j,k} = U_{j,\hat{k}}$. Suppose a country has two regions characterized by the semi-symmetric assumptions of $\theta_{i,k} = \Theta_k$, $i = 1...N-1$ for minority groups and $\theta_{N,k} = M_k = 1 - (N-1)\Theta_k$ for the majority. Assume that the regions are identical except, potentially, in the minority share of the labor force. There is one unique value of γ that permits Θ_k to vary yet still preserves equal utility across regions. Let $U_{Min,k}$ and $U_{Maj,k}$ represent the utility levels of the minority and majority groups in Region k, respectively. Then $U_{Min,1} = U_{Min,2}$ either when $\Theta_1 = \Theta_2$, or when γ satisfies Equation (14).[15]

$$
\begin{aligned}
\gamma &= \left(\frac{\alpha}{(1 - N\Theta_1)\Theta_1^{N-1}M_1 - (1 - N\Theta_2)\Theta_2^{N-1}M_2}\right) \\
&\quad \cdot \left(\frac{\Theta_1^{\sigma}}{(N-1)\Theta_1^{\sigma} + M_1^{\sigma}} - \frac{\Theta_2^{\sigma}}{(N-1)\Theta_2^{\sigma} + M_2^{\sigma}}\right)
\end{aligned}
\tag{14}
$$

3.2 Equilibrium Location Decisions

Under the semi-symmetric assumption, it is straightforward to show that if minorities reside in each region, then the maximum value of y (the weighted average output per worker in the two regions) occurs when the country is perfectly integrated (that is, when the level of diversity is equal in regions one and two so that $\Theta_1 = \Theta_2$). The condition in (14), however, suggests multiple equilibria are possible. Diversity plays an important role in determining individuals' location decisions. Increases in diversity that promote integration will also indirectly increase national productivity. Thus, to understand the economic effects of diversity, we must first identify potential equilibria and determine whether they are stable.

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[15] The economy must also maintain $U_{Maj,1} = U_{Maj,2}$ to ensure that the majority group chooses not to migrate. Simple algebra illustrates that $U_{Min,1} = U_{Min,2}$ implies $U_{Maj,1} = U_{Maj,2}$ as well, so that this second equality restriction is redundant.

3.2.1 Perfect Segregation

In an extreme equilibrium, a country may be perfectly segregated. In this case, the desire of individuals to live among their own group dominates the potential to earn higher wages in other regions. If we continue to assume that a majority exists in each of two regions, then the majority group will compose 100% of one region and 50% of the other. However, members of the majority who remain in the diverse region still have an incentive to migrate. Hence, the model's assumptions become untenable. By removing the assumption that a majority group exists in each region, perfect segregation would then imply that the majority group will compose 100% of one region, and each of the other groups will maintain a population share $\Theta = \frac{1}{N-1}$ in the other region.

An even more realistic assumption is that if groups are choosing to completely segregate, they will continue to do so until each group belongs to one of N exclusive nation-states. In other words, the prior assumption that only two regions exist also becomes inappropriate. By allowing nation-states, individuals will earn $U = \left(\frac{K}{L}\right)\left(\frac{\alpha}{1-\alpha}\right)$ in their own region and $U = 0$ if they choose to move to another region. Thus, this perfectly segregated equilibrium is stable, as no individual has an incentive to migrate. The perfectly segregated outcome may help explain the historical emergence of European nation-states,[16] but is not useful for analyzing migration dynamics since individuals never have an incentive to integrate.

3.2.2 Perfect Integration

The other extreme possibility is that a country is at a perfectly integrated equilibrium in which the minority share of the labor force is equal across regions. If two regions exist, $\Theta_1 = \Theta_2$ implies $U_{Min,1} = U_{Min,2}$ as well. Such an equilibrium is stable only if an exogenous increase in diversity within one region causes minorities to migrate to the other region. That is, if individuals redistribute themselves and return to a perfectly integrated equilibrium.

Intuitively, an increase in diversity will have two potentially opposing effects on minorities. First, they will earn greater utility from living among members of their own group. Unfortunately, however, they may also suffer lower real wages resulting from increased conflict with other groups. Thus, the stability of an integrated equilibrium critically depends upon the sign of $\frac{\partial U_{Min}}{\partial \Theta}$. If minorities experience net gains from diversity, an exogenous increase in diversity within Region 1 will attract additional minorities from Region 2, and perfect integration is unstable. Conversely, integration is stable if the increase in diversity reduces utility and causes minorities to redistribute evenly across regions. Formally, $\frac{\partial U_{Min}}{\partial \Theta} > 0$

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[16] See Alesina, Spolaore, and Wacziarg (2005) for a similar result.

if condition (15) holds.

$$\frac{\partial U_{Min}}{\partial \Theta} > 0 \text{ if} \tag{15}$$
$$\alpha \cdot \sigma \cdot \Theta^{\sigma-1} \cdot M^{\sigma-1} > \gamma \cdot ((N-1)\Theta^\sigma + M^\sigma)^2$$
$$\cdot \left((1 - N\Theta)^2 (N-1) \Theta^{N-2} - N\Theta^{N-1}M\right)$$

Assume $\sigma > 0$ so that labor groups are grossly substitutable and the left hand side of (15) is positive. The right hand side of (15) is guaranteed to be positive if four or more labor groups exist. For the special cases of $N = 2$ or $N = 3$, the right hand side is positive if Θ is small (i.e., if $\Theta < \frac{(2N-1)N-\sqrt{(5N-4)N}}{2(N^2-1)N} \equiv \tilde{\Theta}$).[17] Since production is log-linear in γ, Equation (16) can identify the unique value of interaction costs ($\tilde{\gamma}$) that will determine whether an increase in diversity will attract additional minorities to migrate. That is, Equations (15) and (16) imply that $\frac{\partial U_{Min}}{\partial \Theta} > 0$ and perfectly integrated equilibria are unstable if one of the three conditions in (17) is satisfied.

$$\tilde{\gamma} \equiv \left(\frac{\alpha \cdot \sigma \cdot \Theta^{\sigma-1} \cdot M^{\sigma-1}}{((N-1)\Theta^\sigma + M^\sigma)^2}\right) \tag{16}$$
$$\cdot \left(\frac{1}{\left((1-N\Theta)^2 (N-1) \Theta^{N-2} - N\Theta^{N-1}M\right)}\right)$$

$$
\begin{aligned}
(1) \; \gamma &< \tilde{\gamma} \text{ and } N \geq 4 \\
(2) \; \gamma &< \tilde{\gamma} \text{ and } N \leq 3 \text{ and } \Theta \leq \tilde{\Theta} \\
\text{or } (3) \; \gamma &> \tilde{\gamma} \text{ and } N \leq 3 \text{ and } \Theta > \tilde{\Theta}
\end{aligned} \tag{17}
$$

The intuition behind this result is simple. Minorities experience net utility gains from increases in diversity so long as the costs of interaction and associated wage penalties remain low. Note that condition (3) implies that if few groups exist and the minority population share is already high, diversity will augment utility regardless of the size of interaction costs (since $\tilde{\gamma} < 0$ in this case).

The results indicate that perfectly integrated equilibria are stable only if exogenous interaction costs (γ) are high. High costs imply large wage penalties for living in diverse regions. Thus, if one region experiences a rise in diversity, groups will redistribute themselves evenly across regions. This suggests an interesting counter-intuitive result: since high interaction costs promote integration, and perfectly integrated equilibria produce the maximum level of

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[17]Note that $\tilde{\Theta}$ exceeds the maximum value of diversity possible if a majority group exists when $N > 3$.

aggregate productivity, an increase in interaction costs may be able to promote productivity. Section 3.2.4 explores this possibility in greater detail.

3.2.3 Partial Segregation

Equation (14) identifies the level of interaction costs that permits a partially segregated equilibrium such that $U_{Min,1} = U_{Min,2}$ and $U_{Maj,1} = U_{Maj,2}$, but the level of diversity differs across regions (that is, $\Theta_1 \neq \Theta_2$). Define this value of γ as $\gamma^{Seg}(\Theta_1, \Theta_2)$. Equation (15) suggests that interaction costs will also play a fundamental role in determining the stability of the equilibrium.

Assume (for simplicity) that $N > 3$ groups exist so that Condition (1) among the inequalities in (17) determines the migration decisions of individuals. Then $\tilde{\gamma}$ (from Equation (16)) represents the threshold such that if diversity costs exceed this value, minorities will be attracted to the less diverse region. Since $\tilde{\gamma}$ is a function of Θ, migration decisions for partially segregated societies will depend upon the relative values of $\tilde{\gamma}$ in Regions 1 and 2. Stability of equilibria requires that migration decisions are insensitive to the region in which an exogenous diversity shock occurs, and is determined by the size of the observed level of interaction costs, $\gamma^{Seg}(\Theta_1, \Theta_2)$, relative to $\tilde{\gamma}(\Theta_1)$ and $\tilde{\gamma}(\Theta_2)$. Table 1 summarizes possible outcomes.

Table 1: Stability of Partially Segregated Equilibria Assuming that Diversity is Greater in Region 2.

	Scenario	Stability of Partial Segregation	Migration Consequence of Increased Diversity
(1)	$\tilde{\gamma}(\Theta_1) < \gamma^{Seg}(\Theta_1, \Theta_2) < \tilde{\gamma}(\Theta_2)$ $\uparrow \Theta_1 \rightarrow$ Minorities attracted To Region 2 $\uparrow \Theta_2 \rightarrow$ Minorities attracted To Region 2	Stable	Segregation
(2)	$\tilde{\gamma}(\Theta_1) > \gamma^{Seg}(\Theta_1, \Theta_2) > \tilde{\gamma}(\Theta_2)$ $\uparrow \Theta_1 \rightarrow$ Minorities attracted To Region 1 $\uparrow \Theta_2 \rightarrow$ Minorities attracted To Region 1	Stable	Integration
(3)	$\tilde{\gamma}(\Theta_1) > \gamma^{Seg}(\Theta_1, \Theta_2) < \tilde{\gamma}(\Theta_2)$ $\uparrow \Theta_1 \rightarrow$ Minorities attracted To Region 1 $\uparrow \Theta_2 \rightarrow$ Minorities attracted To Region 2	Unstable	$\uparrow \Theta_1 \rightarrow$ Integration $\uparrow \Theta_2 \rightarrow$ Segregation
(4)	$\tilde{\gamma}(\Theta_1) < \gamma^{Seg}(\Theta_1, \Theta_2) > \tilde{\gamma}(\Theta_2)$ $\uparrow \Theta_1 \rightarrow$ Minorities attracted To Region 2 $\uparrow \Theta_2 \rightarrow$ Minorities attracted To Region 1	Unstable	$\uparrow \Theta_1 \rightarrow$ Segregation $\uparrow \Theta_2 \rightarrow$ Integration

Assume Region 2 is more diverse so that $\Theta_1 < \Theta_2$. If Region 1 experiences an exogenous increase in diversity and $\tilde{\gamma}(\Theta_1) < \gamma^{Seg}(\Theta_1, \Theta_2)$, Equation (15) implies that mi-

norities will earn less utility in Region 1, and therefore desire to move to Region 2. Similarly, minorities will move to Region 2 if diversity exogenously increases in Region 2 and $\gamma^{Seg}(\Theta_1, \Theta_2) < \tilde{\gamma}(\Theta_2)$ since observed interaction costs are lower than the threshold established in (16). The equilibrium is stable since migration responses do not depend upon which region experiences an exogenous diversity shock. Therefore, partially segregated equilibria will be stable and increases in diversity will induce the economy to become more segregated if $\tilde{\gamma}(\Theta_1) < \gamma^{Seg}(\Theta_1, \Theta_2) < \tilde{\gamma}(\Theta_2)$ and $\Theta_1 < \Theta_2$. Stable equilibria also occur when $\tilde{\gamma}(\Theta_1) > \gamma^{Seg}(\Theta_1, \Theta_2) > \tilde{\gamma}(\Theta_2)$ and $\Theta_1 < \Theta_2$. In this case, however, diversity shocks attract minorities to the less diverse Region 1. Thus, diversity would encourage integration.

Partially segregated equilibria are also possible. Suppose $\tilde{\gamma}(\Theta_1) > \gamma^{Seg}(\Theta_1, \Theta_2) < \tilde{\gamma}(\Theta_2)$ and $\Theta_1 < \Theta_2$. Low values of $\gamma^{Seg}(\Theta_1, \Theta_2)$ will attract minorities to the region experiencing a diversity shock. If the shock occurs in Region 1, the country will integrate. If it occurs in Region 2, the country will segregate. The results are opposite from the unstable equilibria defined by high interaction costs, $\tilde{\gamma}(\Theta_1) < \gamma^{Seg}(\Theta_1, \Theta_2) > \tilde{\gamma}(\Theta_2)$, since high costs imply that minorities will leave the region experiencing the shock.

In principle, any of the four scenarios in Table 1 could arise. A more useful exercise is to ask which scenario is likely. First, unstable equilibria are unsustainable in the long run. Instead, assume that the economy is at a stable partially segregated equilibrium. The relevant question becomes whether or not $\tilde{\gamma}(\Theta_1) > \tilde{\gamma}(\Theta_2)$ when $\Theta_1 < \Theta_2$. If so, then diversity leads to integration and indirectly generates productivity gains. This condition holds when the expression in (18) is greater than one.

$$
\begin{aligned}
\frac{\tilde{\gamma}(\Theta_1)}{\tilde{\gamma}(\Theta_2)} =\ & \left(\frac{\Theta_1^{\sigma-1} \cdot M_1^{\sigma-1}}{\Theta_2^{\sigma-1} \cdot M_2^{\sigma-1}} \right) \\
& \cdot \left(\frac{(N-1) \cdot \Theta_2^{\sigma} + M_2^{\sigma}}{(N-1) \cdot \Theta_1^{\sigma} + M_1^{\sigma}} \right)^2 \\
& \cdot \left(\frac{(1 - N \cdot \Theta_2)^2 (N-1) \Theta_2^{N-2} - N \cdot \Theta_2^{N-1} M_2}{(1 - N \cdot \Theta1)^2 (N-1) \Theta_1^{N-2} - N \cdot \Theta_1^{N-1} M_1} \right)
\end{aligned}
\tag{18}
$$

Equation (18) approaches infinity as Θ_1 approaches 0, and it equals one when $\Theta_1 = \Theta_2$. The function does not monotonically decrease, and many cases exist such that $\frac{\tilde{\gamma}(\Theta_1)}{\tilde{\gamma}(\Theta_2)} < 1$. However, if $N > 3$ groups exist, the more diverse region has a population size equal to or larger than the other region (i.e., $L_1 \leq L_2$ when $\Theta_1 < \Theta_2$), and national diversity is limited to ensure that a majority group exists in all regions, this is sufficient (though not necessary) to guarantee that $\tilde{\gamma}(\Theta_1) > \tilde{\gamma}(\Theta_2)$ when $\Theta_1 < \Theta_2$.[18] That is, it is likely that

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[18]The numerator of $\frac{\tilde{\gamma}(\Theta_1)}{\tilde{\gamma}(\Theta_2)}$ monotonically decreases in Θ_1, while the denominator is sure to increase in Θ_1

$\tilde{\gamma}(\Theta_1) > \tilde{\gamma}(\Theta_2)$ when $\Theta_1 < \Theta_2$ and that diversity causes partially segregated countries to integrate.[19] Since integration bolsters productivity, diversity will indirectly increase national output per worker.

3.2.4 Interaction Costs and Partially Segregated Equilibria

Legislators may be less interested in the consequences of diversity and more interested in the effects that policy changes may have on productivity. In particular, governments may have some control over a country's institutions that influence the cost of interaction between groups at a given level of diversity, γ. Diversity costs directly reduce output in the production function. However, since γ exacerbates productivity losses for diverse societies, changes in γ will also have consequences for individuals' location decisions and therefore indirectly affect productivity.

Suppose an economy is at equilibrium such that $U_{Min,1} = U_{Min,2}$.[20] The difference in utility generated by an increase in the cost parameter γ, imposing the semi-symmetric assumption, is given by Equation (19).

$$\frac{\partial(U_{Min,1} - U_{Min,2})}{\partial\gamma} = \left(\frac{K}{(1-\alpha)L}\right) \cdot \tag{19}$$
$$\left((1 - N\Theta_2)\Theta_2^{N-1}M_2 - (1 - N\Theta_1)\Theta_1^{N-1}M_1\right)$$

An increase in the cost parameter does not affect the difference in utility between Regions 1 and 2 if the country is perfectly integrated (i.e., if $\Theta_1 = \Theta_2$). However, the increase has strong implications for partially segregated equilibria. Suppose more minorities reside in Region 2 than in Region 1 so that $\Theta_1 < \Theta_2$. Then an increase in diversity costs will attract minorities to Region 1 and therefore cause the country to integrate if utility in Region 1 becomes larger than that of Region 2 and Inequality (20) is satisfied.

$$\frac{\partial(U_{Min,1} - U_{Min,2})}{\partial\gamma} > 0 \text{ if} \tag{20}$$
$$\frac{(1 - N\Theta_2)\Theta_2^{N-1}M_2}{(1 - N\Theta_1)\Theta_1^{N-1}M_1} > 1$$

---

if $\Theta_1 \leq \frac{N-2}{N\cdot(N+1)}$. Restricting $\bar{\Theta} \leq \frac{1}{2\cdot(N-1)(1+\lambda)}$, where $\lambda = \frac{L_2}{L_1}$, ensures that $\Theta_1 = 0$ when the majority group maintains 50% of the labor force in Region 2. If imposed, the maximum value of $\Theta_1 = \frac{1}{2\cdot(N-1)(1+\lambda)}$. This value is certain to be less than $\frac{N-2}{N\cdot(N+1)}$ if $N > 3$ and $\lambda > 1$.

[19] This theoretical result also appears to be true in practice. The minority share of US employment increased from 13.2% in 1970 to 25.5% in 2000, while the coefficient of variation across the continental US states declined from 0.78 to 0.60. Similarly, the share of college-educated workers grew from 7.9% to 18.4%, and the coefficient of variation decreased from 0.72 to 0.57.

[20] Also, $U_{Maj,1} = U_{Maj,2}$.

The left hand side of Inequality (20) equals one when $\Theta_1 = \Theta_2$ and is monotonically increasing in Θ_2 if $N \geq 4$ or $\Theta_2 \leq \tilde{\Theta}$. Thus, these conditions are sufficient (but not necessary) for ensuring that an increase in interaction costs will promote integration. Note the similarity between this result and the stability of equilibria identified in (17). Together, the conditions imply that low levels of interaction costs (γ) or diversity (Θ) will not permit stable perfectly integrated equilibria, but that increases in interaction costs will encourage groups to integrate. This seemingly counter-intuitive result becomes more obvious when considered in the context of wages paid to minority workers. Firms pay minorities progressively lower wages as their labor force participation rises. This cost of working with own-group members becomes especially burdensome as conflict costs increase, since conflict disproportionately hurts diverse societies. High costs, therefore, push minorities to seek better wages in regions dominated by the majority. Members of the majority group operate in much the same way. This migration occurs until costs are sufficiently high so that individuals are unwilling to accept any additional wage reduction and therefore choose to perfectly integrate.

Since maximum productivity levels occur at perfectly integrated equilibria, Inequality (20) generates the ironic conclusion that exogenous increases in the cost of interaction between groups can indirectly bolster productivity by encouraging groups to integrate. Conversely, efforts to reduce inter-group friction may simply cause individuals to segregate, thereby indirectly reducing productivity. While interaction costs are certain to reduce output for perfectly integrated societies, the effects are more nuanced for partially segregated ones.

4   Simulated Economies

Structural estimation of the production function in Equation (9) is not possible since closed-form solutions for σ do not exist. Reduced form estimation of the effects of diversity should also account for segregation since diversity will generate differing consequences for partially segregated and perfectly integrated countries. In the absence of reliable cross-country segregation data, however, numerical analysis is required. The following subsections improve understanding by simulating production for various parameter values.

4.1   Simulated Production

This subsection estimates the direct effects of diversity by simulating production according to parameter values that characterize the college-educated labor force of the United States in 2000. Assume labor's share of income, α, equals 2/3. In 2000, Whites represented
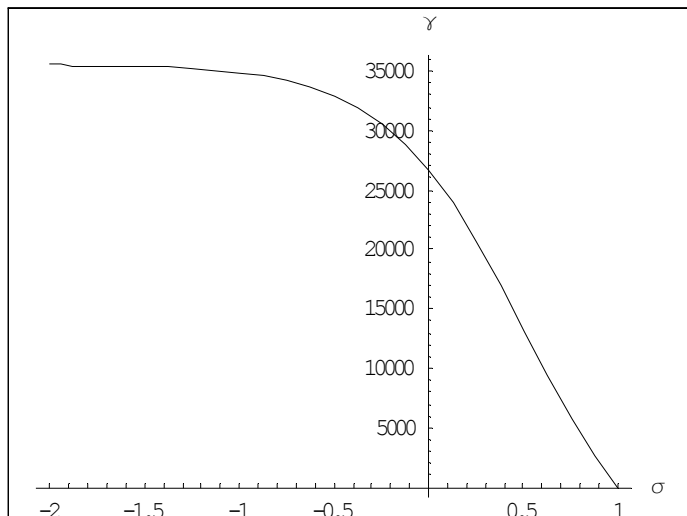
Figure 2: Maximum γ. Assume $N = 5$, $\alpha = \frac{2}{3}$, and $\Theta = 0.05$. The curve represents the maximum value of γ such that diversity generates productivity gains.

roughly 82% of college-educated employment in the United States, with Asians (6.3%), Blacks (6.2%), Hispanics (4.3%), and Others (1.6%) composing the remaining shares. To closely approximate this demography, I adopt the semi-symmetric assumptions of Subsection 2.1 with $N = 5$ racial groups and $\Theta = 0.05$.[21]

Figure 2 graphs the relationship between the substitutability of labor (σ) and the cost of interaction parameter (γ). The curve represents the maximum value of γ such that an increase in diversity (Θ) generates productivity gains. If the costs of interaction are sufficiently low, or if labor types are highly complementary, an increase in minority group size will augment productivity. Figure 3 demonstrates that marginal effects also critically depend upon the initial level of diversity. The maximum value of γ such that an increase in diversity generates productivity gains is much smaller for large values of Θ.

Since regressions cannot uniquely estimate σ and γ, it is difficult to argue in favor of one combination of the parameter values over another, but the elasticity of substitution between White and Minority workers offers suggestions. Regressions estimate an elasticity of 10 – a large but plausible value.[22] Though the elasticity is high, it still implies that diversity generates economically significant effects.

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[21] As an alternative, simulations could adopt $\Theta = 0.0675$ to characterize the entirety of US employment in 2000. However, the assumption of equal shares across minority groups becomes less tenable, as Asians, Blacks, Hispanics, and Others represent 3.5%, 9.6%, 10.3%, and 2.1% of employment, respectively.

[22] This figure represents the elasticity of substitution between White and Non-White college-educated wage earning males 30 to 50 years old. The elasticity across races for men without a college degree is approximately 20. Given that diversity complements creativity, decision making, and idea generation, one should expect this greater substitutability across races at lower skill levels. See the Appendix for regression details.
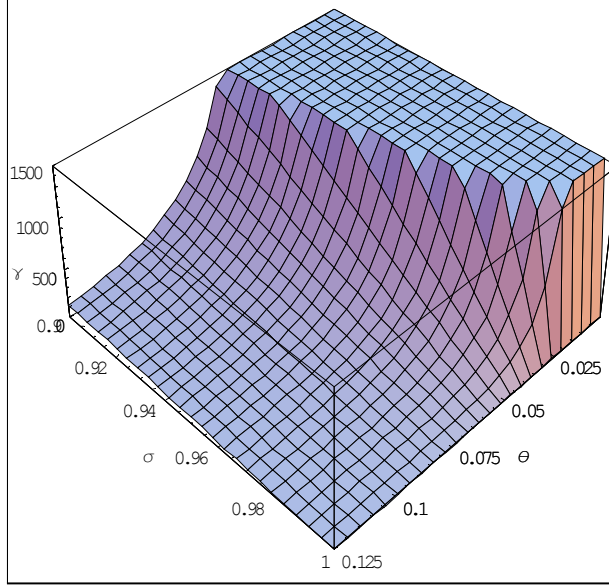
Figure 3: Diversity and Maximum γ. Assume $N = 5$ and $\alpha = \frac{2}{3}$. This graph represents the maximum value of γ such that diversity generates productivity gains. Low existing diversity (Θ) and substitutability of labor (σ) make it more likely that increased diversity will improve productivity.

Figure 4 demonstrates that only low γ values, coupled with σ values near one, can generate high elasticities of substitution.[23] This latter requirement is not surprising – labor types become more substitutable as σ approaches unity, and a change in relative wages forces firms to hire the group that earns lower wages. Less obviously, groups are also more substitutable when γ is small. If conflict is low, firms employ the least expensive workers. If conflict is high, firms force minority workers to take massive pay cuts.

Suppose that $\sigma = 0.95$ and $\gamma = 175$ so that the elasticity of substitution approximately equals 10. Figure 5 graphs output per worker (y) as a function of diversity (Θ). Results are normalized so that $y = 1$ in the absence of diversity.[24] Given the parameter values, maximum productivity occurs when each minority group represents 8.8% of the labor force. Moving from complete homogeneity to this level of diversity would cause output per worker to rise 3.4%. Alternatively, consider that the racial diversity of college-educated workers in the US rose from approximately $\Theta = 0.035$ in 1990 to $\Theta = 0.05$ in 2000.[25] The model implies

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[23] In the special case of $\gamma = 0$, the elasticity of substitution reduces to the typical CES value of $\frac{1}{1-\sigma}$.

[24] Also note that $y = 1$ in the special case when diversity is neutral (i.e., when diversity does not affect productivity).

[25] In 1990, Whites composed 86% of college-educated employment, with Blacks (5.8%), Asians (4.5%), Hispanics (33.3%) and Others (0.3%) representing the remaining shares.
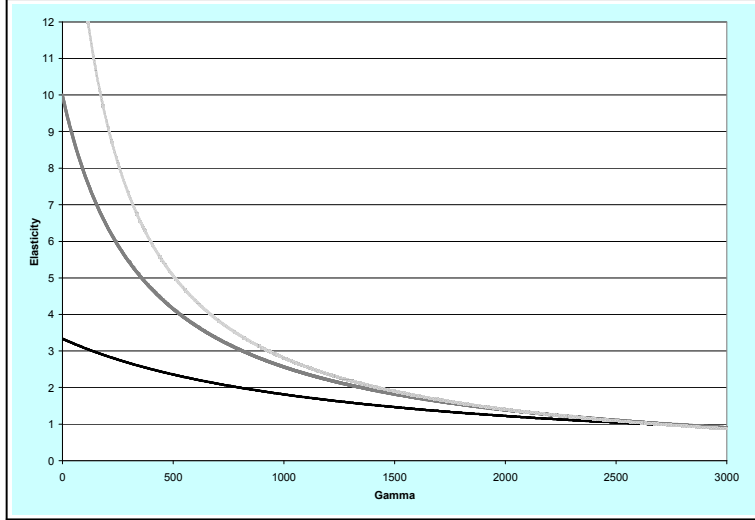
Figure 4: Elasticity of Substitution. Assume $\Theta = 0.05$. The elasticity of substitution between minority and majority-group workers decreases in γ. The light grey, dark grey, and black curves represent $\sigma = 0.95$, $\sigma = 0.9$, and $\sigma = 0.7$, respectively.

that this increase directly caused a 0.6% rise in national productivity.

4.2 Diversity, Conflict Costs, Cross-Region Demography, and National Productivity

The previous subsection simulated the direct effects of diversity assuming that minorities maintain an equal share of employment across regions. However, Section 3 demonstrated that a country can become segregated when parameter values lead to an equilibrium in which two regions have differing levels of diversity. Relative to an even (or perfectly integrated) distribution of diversity, partial segregation will result in reduced macroeconomic productivity. This section adopts a series of diversity (Θ), interaction cost (γ), and labor substitutability (σ) values to gain insight into how diversity and policy decisions influence segregation and thus indirectly affect macroeconomic output per worker.

4.2.1 Diversity, Migration Decisions, and National Productivity

People may choose to forgo wage gains in order to live among members of their own group. The potential existence of such segregation creates a more nuanced relationship between diversity and productivity. Consider an economy with $\sigma = 0.95$, $N = 5$, and $\alpha = \frac{2}{3}$. Assume the level of employment in each region is identical ($L_1 = L_2$) so that the average (or national) level of diversity in Regions 1 and 2 is $\bar{\Theta} = \frac{\Theta_1 + \Theta_2}{2}$. Although Section 4.1
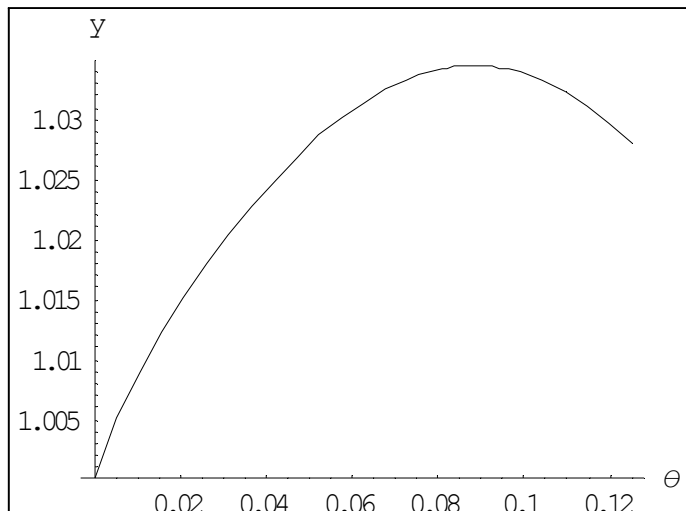
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Figure 5: Diversity and Productivity. Assume $\gamma = 175$ and $\sigma = 0.95$. Maximum productivity occurs when the labor force share (Θ) of each minority group equals 8.8%. In this case, output per worker (y) is 3.4% higher than in the absence of diversity (in which case $y = 1$).

argued for $\gamma = 175$, his level of interaction costs is too low for integration to occur at any level of diversity. Instead, note that each minority group maintained about 6.85% of the college-educated workforce in US states with above-average levels diversity, but just 3.15% in states with below-average diversity.[26] If $\sigma = 0.95$, then $\gamma = 2600$ closely approximates this distribution of workers when $\bar{\Theta} = 0.05$.

Figure 6 illustrates the migration consequences of increased national diversity by display-ing graphs of $U_{Min,1} - U_{Min,2}$ as a function of Θ_2 for several $\bar{\Theta}$ values. (Each graph exhibits odd symmetry around $\bar{\Theta}$ since $\Theta_1 = 2 \cdot \bar{\Theta} - \Theta_2$). Minorities will migrate from Region 2 to Region 1 (and cause Θ_2 to decline) when $U_{Min,1} - U_{Min,2} > 0$. Equilibrium is reached when $U_{Min,1} - U_{Min,2} = 0$. Stability requires that the economy responds to shocks by returning the allocation of groups to its initial position.

The top panel of Figure 6 demonstrates that perfectly segregated equilibria arise when initial levels of national diversity ($\bar{\Theta} = 0.04$) are low. That is, $U_{Min,1}$ is always greater than $U_{Min,2}$, so all minorities move to Region 1. If the total labor force size remains equal across regions (as assumed), the majority group will not completely vacate Region 1 despite having an incentive to do so. Thus, the assumption of equal labor force size becomes untenable. Instead, groups are likely to form nation states in which members from disparate groups do not interact. Any growth among a particular group will be concentrated in one region, and groups never integrate.

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[26]Whites composed 88% of the college-educated workforce in states with below-average diversity, and 74% in states with above average diversity.
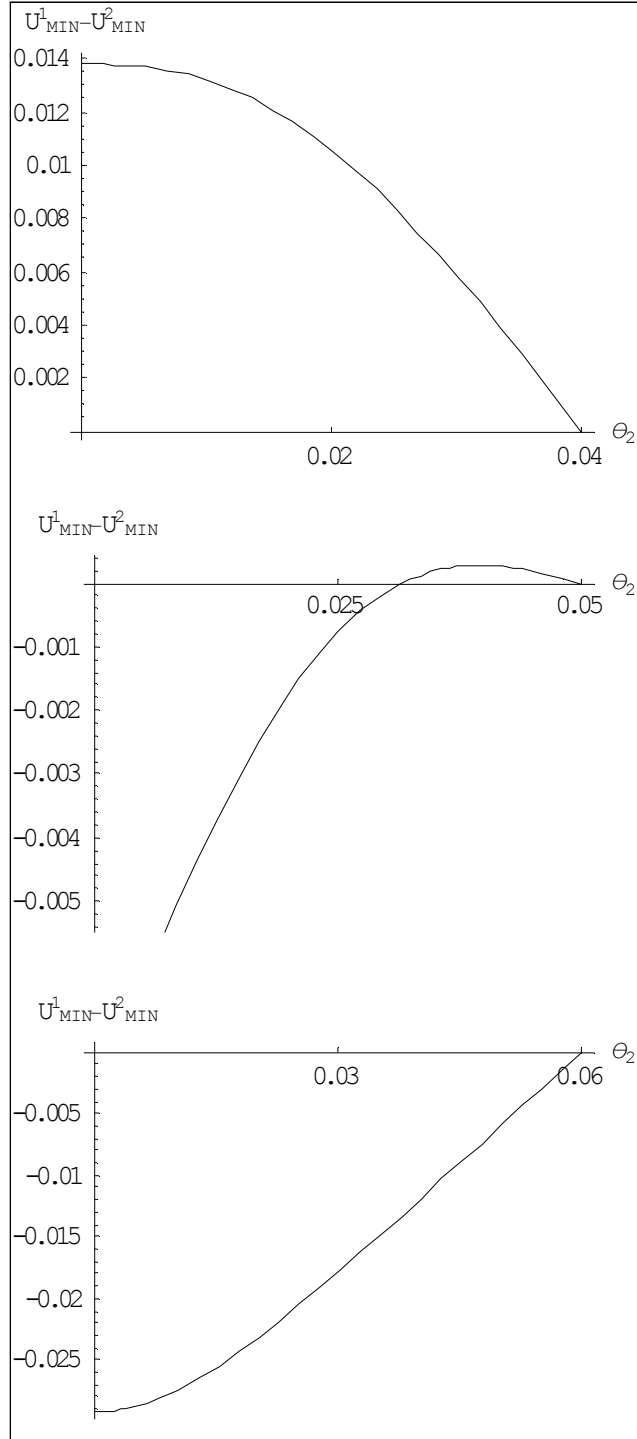
Figure 6: Diversity and Integration. Assume $\sigma = 0.95$, $\gamma = 2600$, $N = 5$, and $\alpha = \frac{2}{3}$. The top, middle, and bottom panels assume and $\bar{\Theta} = 0.04, 0.05$, and 0.06, respectively. Minorities will migrate from Region 2 to Region 1 and Θ_2 will decrease when $U_{Min,1} - U_{Min,2} > 0$. Low levels of national diversity $(\bar{\Theta})$ promote segregated equilibria, while increased diversity facilitates an even distribution of minorities across regions.

The second panel, in contrast, shows that partially segregated equilibria are possible at higher levels of initial diversity. Suppose that $\bar{\Theta} = 0.05$. Then the equilibrium distribution of workers, $\Theta_1 \approx 0.032$ and $\Theta_2 \approx 0.068$, approximates the college-educated workforce in 2000. Moreover, this equilibrium is stable since $\tilde{\gamma}(0.032) > 2600 > \tilde{\gamma}(0.068)$ as described in Table 1.

Section 3.2.3 demonstrated that continued increases in diversity would induce further integration. If diversity continues to grow, the country will become fully integrated as illustrated in the third panel of Figure 6 ($\bar{\Theta} = 0.06$). Since maximum productivity occurs when a country fully integrates (as discussed in Section 3), diversity can indirectly raise output per worker by reducing segregation. Once a country achieves full integration, parameter values determine whether further increases in diversity are beneficial or detrimental. Gains are likely to persist if conflict costs are low or complementarities are high. These parameters could assume country-specific values. Countries that have established sound institutions that manage diversity well might have low conflict costs. Complementarities could be greater for economies on the technological frontier that rely upon extensive innovation and idea generation.

Figure 7 displays the relationship between the total minority share of the national population and aggregate output per worker assuming $\sigma = 0.95$, $N = 5$, $\alpha = \frac{2}{3}$, $\gamma = 2600$, and $L_1 = L_2$.[27] If the country was perfectly integrated, Equation (11) implies that increases in diversity would reduce output per worker when $\bar{\Theta} > 0.37$. However, utility considerations guarantee that the country will be partially segregated. Diversity induces integration and indirectly raises productivity. By increasing the employment share of each minority group from $\bar{\Theta} = 0.047$ to $\bar{\Theta} = 0.05$ (a modest gain in the total minority share from 18.8% to 20%), integration causes productivity to rise by 4%, and output per worker is 0.6% higher than the level that would occur in a homogenous country.

Segregation plays a clear and important role in determining the net effects of diversity, and economists should control for segregation in empirical analyses. Fortunately for practitioners, the relationship between diversity and productivity remains roughly quadratic. Empiricist may still be able to approximate the effect of diversity without segregation data. This would, however, require that regressions include a cross section of countries with disparate levels of diversity. Furthermore, one should exercise caution in making unqualified statements about the costs and merits of diversity, as the net effects will depend upon relative segregation levels.

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[27]Note that multiple equilibria are possible (i.e., a country may be either fully integrated or segregated, depending upon parameter values and initial conditions). Thus, no functional relationship between diversity and productivity exists.
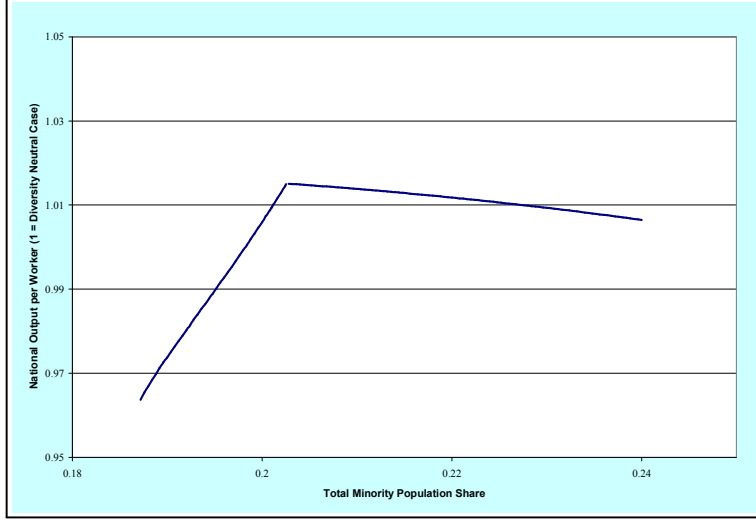
Figure 7: Diversity and National Productivity. Assume $\sigma = 0.95$, $N = 5$, $\alpha = \frac{2}{3}$, $\gamma = 2600$, $L_1 = L_2$, and $\bar{\Theta} = 0.05$. Let $y = 1$ represent the level of output per worker if diversity does not affect aggregate productivity. Output per worker depends upon a country's total minority share of the population, $\left((N-1)\bar{\theta}\right)$, and interregional segregation.

4.2.2 Policy, Migration Decisions, and National Productivity

Section 3.2.4 demonstrated that institutional policy changes that alter the costs of interaction between groups will influence individuals' migration decisions and thus indirectly affect aggregate productivity. Again, closed form solutions for the magnitude of these effects do not exist, so numerical analysis is required. Continue to assume that an economy is characterized by $\sigma = 0.95$, $N = 5$, and $\alpha = \frac{2}{3}$. Also let $\bar{\Theta} = 0.05$ and $L_1 = L_2$. Figure 8 illustrates the migration consequences of policy changes by displaying graphs of $U_{Min,1} - U_{Min,2}$ as a function of Θ_2 for several γ values. As in Figure 6, minorities will migrate from Region 2 to Region 1 (and cause Θ_2 to decline) when $U_{Min,1} - U_{Min,2} > 0$.

The top panel assumes $\gamma = 175$ to match an elasticity of substitution equal to 10 (as advocated in Subsection 4.1). Note that the perfectly integrated equilibrium in which $\Theta_1 = \Theta_2 = 0.05$ is unstable since $175 < \tilde{\gamma}$ from Equation (16). Instead, low interaction costs cause groups to perfectly segregate. That is, $U_{Min,1}$ is always greater than $U_{Min,2}$, so all groups form nation states, and individuals from varied groups will not interact.

The summary in Table 1 indicates that higher levels of interaction costs will permit stable partially segregated equilibria if $\tilde{\gamma}(\Theta_1) < \gamma^{Seg}(\Theta_1, \Theta_2) < \tilde{\gamma}(\Theta_2)$ or $\tilde{\gamma}(\Theta_1) > \gamma^{Seg}(\Theta_1, \Theta_2) > \tilde{\gamma}(\Theta_2)$. The second panel of Figure 8 increases γ to 2600. The perfectly integrated equilibrium remains unstable since $2600 < \tilde{\gamma}$. A slight negative shock to Θ_2 at this equilibrium will make Region 1 more attractive to minorities. Massive migration occurs until
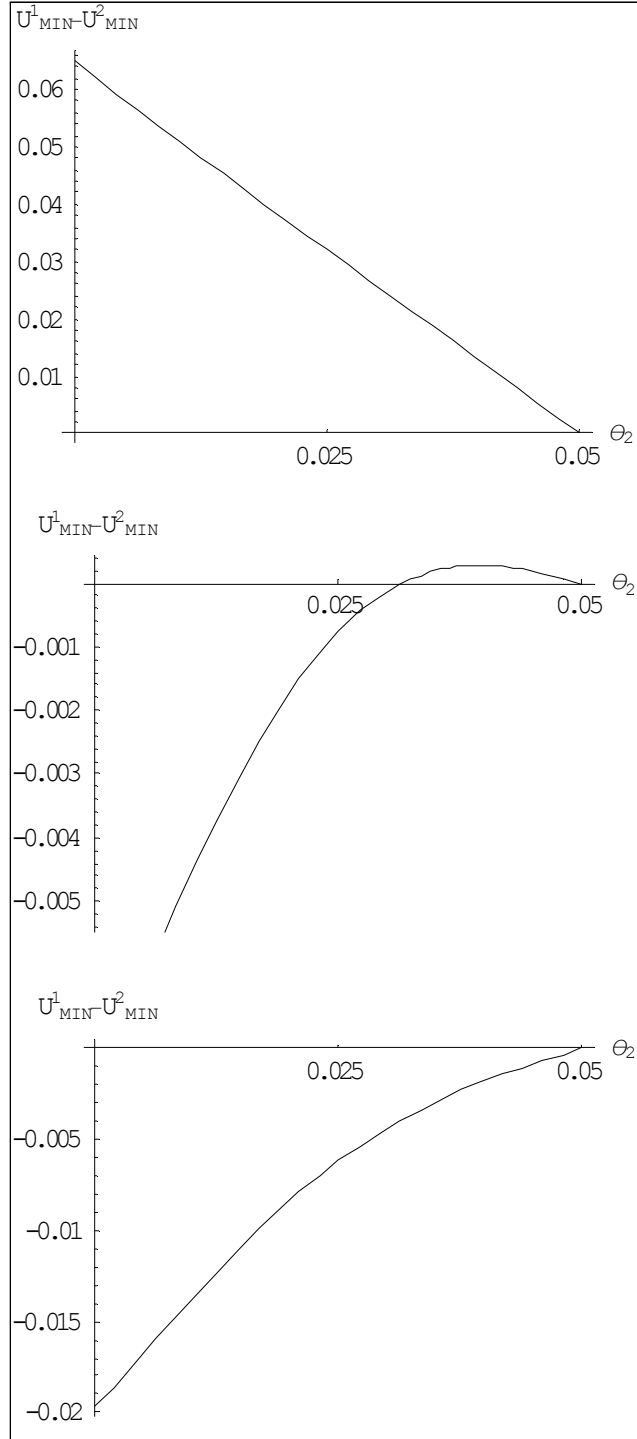
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Figure 8: Conflict Costs and Integration. Assume $\sigma = 0.95$, $N = 5$, $\alpha = \frac{2}{3}$, and $\bar{\Theta} = 0.05$. The top, middle, and bottom panels assume $\gamma = 175, 2600$, and 3000, respectively. Minorities will migrate from Region 2 to Region 1 and Θ_2 will decrease when $U_{Min,1} - U_{Min,2} > 0$. Low γ values promote segregated equilibria, while high γ values ensure an even distribution of minorities across regions.
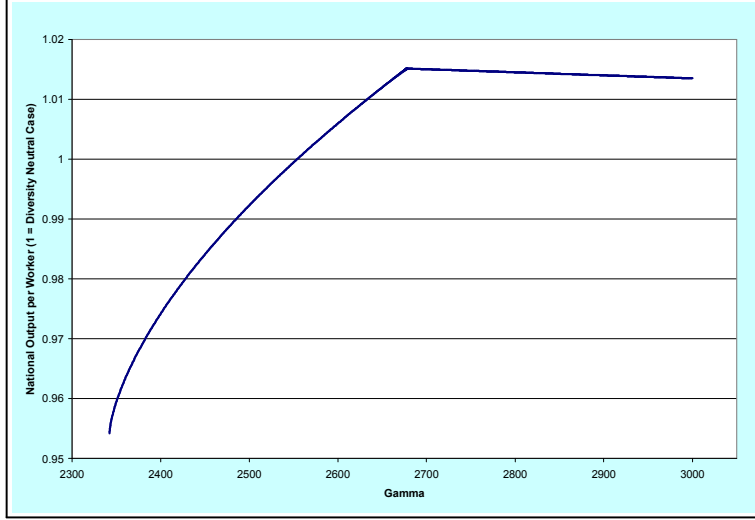
Figure 9: Interaction Costs and National Productivity. Assume $\sigma = 0.95$, $N = 5$, $\alpha = \frac{2}{3}$, $L_1 = L_2$, and $\bar{\Theta} = 0.05$, and let $y = 1$ represent the level of output per worker if diversity does not affect aggregate productivity. Increasing the costs of interaction between groups will induce individuals to integrate, and may therefore raise productivity over a range of values.

$\Theta_1 \approx 0.032$. Section 4.2.2 argued that this partially segregated equilibrium is stable since $\tilde{\gamma}(0.032) > 2600 > \tilde{\gamma}(0.068)$. Graphically, we see that any shock to the relative diversity in Regions 1 and 2 will return the economy to this point.

The final panel in Figure 8 assumes $\gamma = 3000$. High interaction costs eliminate stable partially segregated equilibria and instead encourage individuals to migrate until groups are perfectly integrated across regions. Perfect integration is stable since $3000 > \tilde{\gamma}$ and $N > 3$ (see Section 3.2.2).

Policy changes generate important productivity consequences. First, note that the cost parameter (γ) will not affect perfectly segregated countries (that is, nation states), since members from different groups do not interact with each other. Changes in γ will, however, have large implications in all other scenarios. When γ permits partially segregated equilibria, institutional changes that make interaction between groups more difficult will encourage integration since minorities become less willing to suffer large wage penalties associated with living among members of their own group. This integration stimulates economic activity by increasing the supply of scarce (and complementary) minority groups in less diverse regions. Thus, inter-group communication costs can ironically bolster productivity over a range of values. After the country fully integrates, however, all further interaction costs will diminish productivity.

Figure 9 demonstrates that the effects of such policy changes may be large. If $\sigma = 0.95$,

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$N = 5$, $\alpha = \frac{2}{3}$, $L_1 = L_2$, and $\bar{\Theta} = 0.05$, then an increase in interaction costs that moves society from an extreme distribution in which nearly all minorities live in Region 2 ($\gamma = 2350$, $\Theta_1 = 0.0022$, $\Theta_2 = 0.0978$) to a completely integrated equilibrium ($\gamma = 2680$, $\Theta_1 = 0.05$, $\Theta_2 = 0.05$) will raise output per worker by 5.7%.

5 Conclusions

This paper developed a model of production that includes a term to capture the costs of inter-group communication, allows complementarities between workers of different types, and yet still nests the traditional Solow growth model within it. A simple relationship between intergroup complementarities and costs of interaction determines whether diversity generates direct productivity gains or losses in the absence of labor supply effects. Outcomes become much more complex, however, in the presence of segregation.

Segregation can occur if individuals earn utility from residing among members of their own group. If minority groups and interaction costs are small, groups will form separate enclaves within a country. Members of disparate groups never interact, and nation states arise. If either interaction costs or the minority share of national employment are at moderate levels, however, the country will be partially segregated – diversity varies across regions, but individuals have no incentive to migrate. Increases in diversity or interaction costs generally encourage further integration until all regions of a country maintain equal levels of diversity. Importantly, maximum output always occurs when a country is fully integrated, regardless of whether national diversity generates productivity gains or losses. Thus, when diversity induces integration, it indirectly causes national productivity to rise. Once a country achieves full integration, continued increases in diversity will augment aggregate output only if complementarities are high and interaction costs are low.

This theory of diversity, segregation, and productivity uncovers important policy and empirical ramifications. Politically, it suggests that policy makers should undertake efforts to decrease interregional segregation. If agents earn utility only from real wages and from living among members of their own group, an increase in interaction costs will ironically augment productivity by encouraging groups to integrate. Once a country is fully integrated, however, these costs only serve to reduce output. Thus, other methods of encouraging integration (such as increasing diversity, decreasing own-group preferences, or increasing utility from residing among members of other groups) are likely to be more economically advantageous.

The model also implies that international empirical investigations of diversity will generate misleading results if they do not also consider the role of segregation. Reliable segregation data is not likely to be available at the international level, however. Several solutions to this

limitation exist. First, economists could instead assess the effects of interregional diversity within a country. The model assumes that agents live and work in the same region. Agents need not, however, live and work in the same neighborhood. Thus, within-region segregation should not be as detrimental as cross-regional segregation. Second, valid international accounts of diversity are still possible without segregation data if economists are willing to accept the analyst's assumptions. For example, researchers could assume that interregional segregation doesn't exist, or that all countries are perfectly segregated. Also, panel analyses that assume segregation is constant over time would be appropriate if they employ random (or fixed) effects to control for segregation variation across countries. Finally, quadratic models might provide a close approximation for the effects of diversity, while they would also highlight that some countries benefit from increased heterogeneity and others do not.

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A   Appendix – The Elasticity of Substitution

I employ a decennial dataset covering the contiguous US states from 1980 to 2000 to estimate the elasticity of substitution between Whites and Non-Whites. The Integrated Public Use Microdata Series (IPUMS) by Ruggles et. al. (2004) provides all necessary data to identify a state's labor supply and average wages paid to White and Non-White workers. I then regress relative wages on relative labor supply according to variants of Equation (21).

$$\ln\left(\frac{WhiteWage_{s,t}}{NonWhiteWage_{s,t}}\right) = \alpha + \beta \cdot \ln\left(\frac{WhiteEmp_{s,t}}{NonWhiteEmp_{s,t}}\right) \tag{21}$$

$$+ \sum_{t=1990}^{2000} \delta_t * Decade_{s,t} + \epsilon_{s,t}$$

$$\text{Where } s = 48 \text{ contiguous states, } t = 3 \text{ decades.}$$

$$Wage = \text{Average wage earnings of Whites or Non-Whites.}$$

$$Emp = \text{Employment of Whites or Non-Whites.}$$

$$Decade = \text{Decade indicator variables for 1990 and 2000.}$$

$$\frac{-1}{\beta} = \text{Implied elasticity of substitution.}$$

The regressions in Table 2 use data on wage-earning men, 30 to 50 years old, with a college degree to impute the elasticity of substitution between Whites and Non-Whites. Column 1 displays the results for direct estimation of Equation (21). The coefficient on relative employment, -0.107, implies an elasticity of substitution between Whites and Non-Whites of 9.346. However, relative wages could be endogenous. If a state pays disproportionately more to White workers, it will attract a large number of White workers. Ten year lag values of relative employment can serve as instruments for current-year values. Column 2 demonstrates that controlling for endogeneity decreases the elasticity of substitution to 8.547. Columns 3 and 4 control for the possible effect of diversity on relative wages by including a term measuring the minority share of a state's employment. The coefficient is insignificant. The regression in Column 5 drops observations in 1980 and 1990, and employs a 20 year lag as an instrument for relative employment. This increases the elasticity to 12.346. Altogether, the results suggest an elasticity of substitution between Whites and Non-Whites near 10 for college-educated men in the United States.

If diversity complements creativity, decision making, and idea generation, the elasticity of substitution across races should be lower for high skilled workers than for low skilled labor. Table 3 repeats the specifications displayed in Table 2, but instead estimates the elasticity

Table 2: Elasticity of Substitution. Wage Earning Males, 30-50 Years Old, with a College Degree.

Dependent Variable: ln(Relative Wage)					
	1	**2**	**3**	**4**	**5**
Instruments	None	10 Year Lag	None	10 Year Lag	20 Year Lag
Decades Covered	1980-2000	1980-2000	1980-2000	1980-2000	2000
ln(Relative Employment)	-0.107 (0.031)***	-0.117 (0.038)***	-0.142 (0.045)***	-0.205 (0.062)***	-0.081 (0.027)***
Minority Employment Share			-0.005 (0.006)	-0.012 (0.007)	
Constant	0.422 (0.068)***	0.442 (0.084)***	0.554 (0.165)***	0.767 (0.223)***	0.387 (0.059)***
Implied Elasticity of Substitution	9.346	8.547	7.042	4.878	12.346
Observations	144	142	144	142	48
R-Squared	0.25	0.27	0.26	0.28	0.21

Unit of observation: continental US states.
Relative Wage measured as wages paid to Whites divided
   by wages paid to Non-Whites.
Relative Employment measured as White employees divided
   by Non-White employees.
Note: Only college-educated male employees between 30 and 50
   years old are included in the calculation of state averages.
Instruments: Lag values of ln(Relative Employment).
Cluster-robust standard errors in parenthesis.
*** Coefficient significant at 1%
** Coefficient significant at 5%
* Coefficient significant at 10%
Decade indicator variables suppressed.

of substitution between Whites and Non-Whites for wage earning men who do not hold a college degree. As expected, the estimated elasticity is much higher for low skilled labor, as regressions suggest a value near 20.

Table 3: Elasticity of Substitution. Wage Earning Males, 30-50 Years Old, without a College Degree.

	Dependent Variable: ln(Relative Wage)				
	1	**2**	**3**	**4**	**5**
Instruments	None	10 Year Lag	None	10 Year Lag	20 Year Lag
Decades Covered	1980-2000	1980-2000	1980-2000	1980-2000	2000
ln(Relative Employment)	-0.054 (0.009)***	-0.050 (0.010)***	-0.042 (0.019)**	-0.023 (0.021)	-0.043 (0.012)***
Minority Employment Share			0.001 (0.002)	0.003 (0.002)	
Constant	0.342 (0.021)***	0.332 (0.022)***	0.315 (0.061)***	0.260 (0.068)***	0.349 (0.021)***
Implied Elasticity of Substitution	18.519	20.000	23.810	43.478	23.256
Observations	144	143	144	143	48
R-Squared	0.35	0.35	0.34	0.33	0.31

Unit of observation: continental US states.
Relative Wage measured as wages paid to Whites divided
    by wages paid to Non-Whites.
Relative Employment measured as White employees divided
    by Non-White employees.
Note: Only male employees between 30 and 50 years old without
    college degrees are included in the calculation of state averages.
Instruments: Lag values of ln(Relative Employment).
Cluster-robust standard errors in parenthesis.
*** Coefficient significant at 1%
** Coefficient significant at 5%
* Coefficient significant at 10%
Decade indicator variables suppressed.